

PROCUREMENT WITH BID PREFERENCE & BUYER'S SWITCHING COSTS

THE CASE OF MUNICIPAL BUSES

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Abstract

Switching costs that arise in repeated purchases of durable goods cause buyers to face conflicting incentives: facilitating competition among potential sellers leads to lower prices while restricting competition among them allows buyers to avoid the disruption associated with introducing a new brand. I study this trade-off in an auction environment with bid preference that allows buyers to favor certain sellers. I construct novel data on fleet renewal by municipal bus operators in Poland who use a common format of scoring auctions to implement bid preference. Consistent with their incentive of avoiding switching costs, the operators favor incumbent bus producers. Motivated by this finding, I develop and estimate a structural model of public procurement with bidder favoritism to quantify the main driving forces of the trade-off. Estimates suggest that bid preference programs can balance the trade-off if an auction attracts sufficiently many bidders, whereas forcibly promoting competition while ignoring the underlying lock-in relationship between buyers and incumbent sellers would lead to counter-productive outcomes. Therefore, the design of public procurement should not only target achieving low prices but also account for other aspects contributing to buyers' welfare.

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1 INTRODUCTION

Switching costs arise when buyers repeatedly purchase durable goods, for example, vehicles in a fleet, machinery in production plants, office equipment, or software. Introduction of a new brand may generate costs related to integrating it into existing infrastructure, adding a set of skills to the staff who need to know how to operate, maintain and repair new equipment, and increasing storage capacity for spare parts. These switching costs, also referred to as disruption costs, cause buyers to face conflicting incentives. Facilitating competition among potential sellers leads to lower prices while restricting competition among potential sellers allows buyers to avoid the disruption associated with introducing a new brand.

Proper management of buyers' trade-off between avoiding switching costs and minimizing prices is a task of particular interest from the point of view of public policy when buyers use public funds to cover their expenses. Public procurement of goods and services is a vital part of modern economies, accounting for 10%–20% of GDP in developed countries. Its significance is continuously increasing, as, among others, governments mandate buyers' transition to more environmentally friendly technologies and generously subsidize these efforts. In addition, many buyers use taxpayer money not only to cover purchases but also other expenses related to their economic activity. Finding ways of promoting competition while taking into account buyers' switching costs would allow for a more efficient allocation of public funds.

Auctions are a prevalent format in public procurement due to their allocative efficiency and procedural transparency. In a standard setting, the contract is awarded to the lowest bidder, minimizing the buyer's purchase cost. However, if the identity of the winner matters from the point of view of buyer's welfare, the standard setting may not be the optimal choice. Accommodating bid preference into the auction mechanism is a frequently used refinement of the standard auction format when buyers have additional goals beyond minimizing the cost of purchase. Bid preference allows buyers to favor some bidders over others. Specifically, the buyer assigns weights—known as *bidder preference weights*—to each potential seller before bidding. The winner is a bidder whose effective bid, the product of the price bid and its preference weight, is the lowest. The way the weights affect the ranking of the bids can

be interpreted in terms of assigning bid credits or discounts. For example, 10% bid credit implies that a bid of \$10 is treated as \$11 in the ranking. If it happens to be the winning bid, the buyer pays the original price of \$10.

Auctions with bid preference are a significant part of public procurement. In 2006, 20% of the dollar value of U.S. federal government procurement was awarded to favored bidders (Krasnokutskaya and Seim, 2011). Bid preference is also present in many business-to-business settings, including so-called *Tier II* programs used by large private businesses, including Chevron, Coca-Cola, Microsoft, and MillerCoors (Mummalaneni, 2022). Typically, preferential treatment is given to sellers who may be disadvantaged in some way compared to the competitors, including local or domestic firms, small entrepreneurs, and women- or minority-owned businesses.

I introduce a different possible rationalization for bid preference in procurement auctions. Since it affects the relative chances of winning by particular bidders, bid preference may allow buyers facing switching costs to account for both sides of their trade-off in the auction design. By favoring incumbent sellers, buyers increase the incumbents' chances of winning and provide incentives for non-incumbents to bid more aggressively, mitigating the upward pressure on prices. However, if participation in auctions is costly for bidders, too much discrimination may discourage non-incumbents from submitting a bid. If this is the case, bid preference fails to balance the buyer's trade-off, becoming a tool for stifling the competition and allowing incumbents to enjoy monopolistic rents.

The main goal of this paper is to understand the challenges resulting from a lock-in relationship between buyers and incumbent providers and propose solutions to improve the efficiency of public procurement under the presence of disruption costs. To achieve this goal, I study the behavior of Polish public municipal bus operators who repeatedly procure new buses via auctions with bid preference. I construct a novel dataset on auction design linked to operator's fleets and estimate a structural model to quantify the main driving forces of the trade-off: producers' costs of participation in an auction and buyers' disruption costs associated with the introduction of a new brand to the fleet. Lastly, I use the estimates to perform counterfactual exercises to assess whether bid preference programs can balance

buyers' trade-off between minimizing price and avoiding switching costs and suggest policies targeting improvement in buyers' welfare.

The case of Polish municipal buses is particularly suitable for studying the trade-off. Bus operators are likely facing switching costs related to the introduction of a new brand to their fleet. Even though they are not legally allowed to openly favor or discriminate against potential bidders, they implement bid preference implicitly using a prevalent format of scoring auctions¹. In scoring auctions, bids are ranked according to the scoring rule announced by the buyer before bidding and containing a list of scoring criteria assigning points in many dimensions of the demanded order. The choice of scoring criteria remains at the buyer's discretion and the operators tend to choose criteria related to bus technological solutions that are specific to particular bus producers. I show that the scoring auction designed in this way becomes effectively an auction with bid preference. Using the official auction specifications from over 1000 auctions organized between 2006 and 2022, I reconstruct bidder preference weights implied by the number of points assigned to the potential bidders in criteria related to bus technological solutions and link them to the evolution of operators' fleets. Moreover, the institutional environment creates substantial barriers to entry for new bus producers, limiting fringe bidding and enabling me to identify the set of potential bidders in every auction. Lastly, operators renew only a fraction of their fleet at a time, which allows me to follow changes in operator's favoritism towards particular bidders in a response to changes in the fleet and distill the effects of switching costs from the effects of unobserved qualities of a match in operator-producer pairs.

Data suggest that operators systematically favor incumbent producers, consistent with their motive to avoid switching costs. In turn, producers who show up in auctions are mainly the favored ones. Guided by these observations, I develop and estimate a structural model of interactions between a buyer and bidders via auctions with bidder preference weights. The model builds on the empirical auction literature ([Krasnokutskaya and Seim, 2011](#); [Krasnokutskaya, 2011](#); [Athey, Levin, and Seira, 2011](#)) and adapts it to the specifics of

¹This auction format is pro-actively promoted in European Union (directives 2014/24/EU and 2014/25/EU), and some industries are required to use it in their purchases. As a result, 60% of all auctions are scoring auctions. They are also present in the US, for example in a form of A+B auctions for highway construction ([Lewis and Bajari, 2011](#)).

my application. Specifically, it encompasses three stages: the operator choosing the degree of favoritism towards potential bidders, potential bidders choosing whether to pay an entry cost and participate in an auction, and actual bidders submitting their bids. Since bidder preference weights are implemented via scoring rule criteria related to producers' technology, the weights intended by the operators may not be the same as the weights actually used to rank the bids due to (possibly small) technology shocks. To account for this, I introduce a random disturbance into the process of assigning bidder preference weights, generalizing the standard model of the auction with bid preference. The data-generating process implies arbitrary patterns of bidder preference weights at the auction level, which constitutes a significant departure from the data environment usually exploited in the literature (Hubbard and Paarsch, 2014) and affects the characterization of bidding equilibrium (Lebrun, 2006). I adapt the numerical method of solving for optimal bidding to account for arbitrary patterns of bidder preference weights and the associated phenomenon of bid separation (Hubbard and Kirkegaard, 2019). Lastly, I take advantage of the richness of my data and use a wide range of fleet variables to measure the impacts of various aspects of switching costs.

The estimated average entry costs amount to between 8.4% and 10.3% of the project completion cost and significantly discourage participation, particularly among unfavored potential bidders. The results suggest the presence of a two-sided lock-in relationship between operators and producers. Operators' disruption costs are related both to introducing a new brand to their fleet and maintaining higher fleet diversity. Operators are willing to pay 28.2% of the average order value for a win of the most suitable incumbent. They also tend to increase the degree of discrimination with an increase in the number of distinct brands in their fleets. On the producer side, the lock-in manifests itself in a form of an incumbent advantage that originates from already-established business relationships between an operator and a producer. Incumbent advantage decreases incumbents' entry costs making it *easier* for them to participate in auctions regardless of bid preference.

The main result of this paper shows that one can increase the efficiency of public funds allocation by accounting for other aspects of buyers' welfare in addition to targeting low prices while designing procurement markets. Counterfactual experiments confirm that bid preference programs can balance the trade-off between price and switching costs by reallo-

cating winning probability towards the favored bidders. A necessary condition for this to happen is that the auctioneer manages to attract sufficiently many bidders, which may be difficult to achieve because of the incumbent advantage in entry costs as well as a relatively low number of potential bidders. In turn, forcibly promoting competition while ignoring the underlying lock-in relationship between buyers and incumbent sellers would have a detrimental effect on buyers' welfare. It is often the case that benefits from lower prices are offset by an increase in costs associated with excessive switching. A policy improving the efficiency of public procurement under the presence of disruption costs should both encourage competition and address the lock-in relationship between operators and incumbent producers. I discuss several policies based on subsidizing potential bidders' entry, market entry of new potential bidders, and expanding the offer of existing potential bidders, as well as redesigning the timing and scale of organized auctions.

The remainder of this paper is structured as follows. Section 2 summarizes related literature. Section 3 describes the market for purchasing new buses by public municipal bus operators in Poland. In section 4, I introduce data and discuss the main patterns. Section 5 introduces the theoretical model. Section 6 discusses estimation strategy to recover primitives of the model. Section 7 presents estimation results. In section 8 I use the estimated model to perform counterfactual exercises. Section 9 discusses potential policy solutions to improve the efficiency of public procurement in presence of switching costs. Section 10 concludes.

2 LITERATURE REVIEW

This paper contributes to a few strands of literature, concerning buyer's switching costs, favoritism in auctions, scoring auctions, asymmetric auctions, and low bidder participation in public procurement procedures.

Buyer's switching costs are extensively studied in the literature (Klemperer, 1995). More recent applications concern consumer heterogeneity and inertia (Hortaçsu, Madanizadeh, and Puller, 2017; Miller, Petrin, Town, and Chernew, 2019), transaction costs (Luco, 2019), and interactions between adverse selection, regulation and inertia (Polyakova, 2016). A

common denominator of these papers is that they study switching costs in posted price environments when agents have full discretion over their choices. [Cabral and Greenstein \(1990\)](#) and [Greenstein \(1993\)](#) showed that government purchases may also be affected by switching costs. To the best of my knowledge, this paper is the first study to provide a comprehensive analysis of buyers' switching costs within the auction environment by identifying trade-offs and suggesting solutions to improve market efficiency.

Several authors studied applications of bid preference programs that were designed to support domestic or local firms ([McAfee and McMillan, 1989](#); [Rosa, 2019](#)), minority and women-owned businesses ([Ayres and Cramton, 1996](#); [Mummalaneni, 2022](#)), and small entrepreneurs ([Krasnokutskaya and Seim, 2011](#)). Even though in these papers bidder discrimination serves normative purposes, favoring high-cost potential bidders may lead to a decrease in procurement costs ([McAfee and McMillan, 1989](#); [Corns and Schotter, 1999](#)). My paper shows that the use of bidder preference weights can be rationalized by a positive motive, to avoid switching costs of introducing a new brand. I extend the analysis of buyer's welfare in the auction context, by showing that it does not only depend on procurement costs (prices) but also on other types of buyer's costs. I also generalize the standard approach in modeling bidder favoritism by introducing randomness into the bidder preference weights chosen by the auctioneer.

In papers studying favoritism in auctions, bidder discrimination is legally sanctioned by the government. In turn, literature on scoring auctions ([Che, 1993](#); [Asker and Cantillon, 2008, 2010](#)) focuses on quality aspects of the order. The sellers may receive a better score (and hence have larger chances of winning) by offering higher quality products, which may be costly for them. In principle, each potential bidder may ex-ante receive the highest score. My paper builds a bridge between these two strands of literature by showing that when open discrimination is not allowed, bid preference programs can be implemented implicitly through scoring auctions. This is done by choosing scoring criteria that make it ex-ante impossible for some potential bidders to receive the full score.

The existing papers studying bidder preference weights consider a setting in which there are two groups of bidders—preferred and non-preferred. The bid preference rate is set by the market regulator instead of the buyers, and remains the same for all bidders within

a group. My application is substantially different, as buyers assign arbitrary weights to potential bidders. This feature makes analysis more challenging. I show that the auction with arbitrary patterns of bidder preference weights is equivalent to asymmetric auctions with varying support of project completion cost distribution and use results of [Lebrun \(2006\)](#) to show the existence and uniqueness of equilibrium in the bidding game and provide a characterization of the equilibrium bidding. The characterization implies the possibility of bid separation, which requires the adaptation of numerical methods to solve for optimal bidding ([Hubbard and Kirkegaard, 2019](#)). [Bolotnyy and Vasserman \(2021\)](#) account for one possible bid bifurcation point. I adapt existing numerical methods of solving for equilibrium bidding to account for arbitrary patterns of bid bifurcation that may occur due to arbitrary bidder preference weights.

It is a well-documented fact that public procurement procedures often attract few bidders. Over 24% of all auctions in EU observe a single bidder ([Titl, 2021](#)), and 45% of the value of federal contracts has been assigned in a single bidder setting in the US ([Kang and Miller, 2022](#)). Low participation is attributed to incumbent’s cost advantage ([Iossa, Rey, and Waterson, 2022](#)), insufficient publicity and information about the contracts ([Coviello and Mariniello, 2014](#)), corruption and political connections ([Schoenherr, 2019](#); [Baranek and Titl, 2020](#); [Decarolis, Fisman, Pinotti, and Vannutelli, 2020](#)), natural monopoly. This paper adds to this list by showing that low participation can also be attributed to buyers’ attempts to avoid switching costs of introducing a new brand.

3 PROCURING CITY BUSES IN POLAND

This section describes the market environment in which Polish municipal bus operators procure new buses and shows how they use the scoring auction format to implement bid preference.

3.1 MARKET ENVIRONMENT

Poland is a relatively big market for municipal buses. At each point in time, there are around 10 producers who supply operators with new buses. The exact numbers vary with

producers' entry and exit. The producers are usually well-established, regular participants in the market. High producer's entry costs, including obligatory vehicle certification and setting up a nationwide system of affiliated workshops, limit significantly fringe bidding. They sell their products to approximately 200 municipal operators, holding together a stock of over 12,000 vehicles. Operators have buses of different ages and renew their fleet gradually through small orders. The vast majority of bus operators are either owned or financed by local authorities. That imposes a legal requirement of using public procurement for any larger purchases, to ensure transparency of public fund spending. The procurement format in use is the scoring auction, mandated by national and European EU procurement laws.

An auction is announced when the operator publishes an auction specification online, which is a set of documents specifying order requirements. It also includes a definition scoring rule, which is an algorithm that serves to rank submitted bids. The scoring rule contains a list of criteria related to the order that are preferred but not required by the operator. Each criterion assigns several points to a bid that satisfies it. For each bid, points from different criteria are added up to a maximum of 100 points that an offer may receive. The winner is a bidder whose offer gets the largest number of points.

After the announcement, potential bidders are given 1–2 months to prepare their bids. During this time they are allowed to ask the operator questions regarding specifics of the order. The dialog between potential bidders and the operator is published together with the auction specification. Despite questions being asked anonymously, they often refer to particular technological solutions which are specific to some producers. Next, bidders submit their bids in secret. The bids are opened and ranked by an operator using the scoring rule. Eventually, the winner is announced. The operator publishes auction results including the total number of points assigned through the scoring rule for each of the bidders.

3.2 SCORING AUCTIONS AND BIDDER PREFERENCE WEIGHTS

Bidder preference weights $\theta = \{\theta_j\}_j$ is a vector of positive numbers, one for each of the potential bidders. Operators assign them to favor or discriminate against bidders. Bidder preference weights affect the ranking of submitted bids. The winner is a bidder who submits

the lowest value of a product of bid and the weight:

$$j \text{ wins} \Leftrightarrow \arg \min_k \theta_k b_k$$

where b_k is a price submitted by bidder k . Therefore, bidder preference weights give an ex-ante price advantage to preferred bidders. It is no longer necessary that the winner is a bidder who submits the lowest price. Bidder preference weights require normalization. For convenience, I normalize the lowest θ_j – the weight assigned to the most preferred bidder – to 1.

Polish bus operators are not legally allowed to openly discriminate against potential bidders. However, Instead, the commonly used scoring auction format creates an opportunity to implement bidder preference weights implicitly. A key element of the environment is the scoring criteria used for bid evaluation. Scoring criteria can be divided into two groups: criteria related to the bus and criteria related to the offer (but not directly to the offered bus itself). The latter includes factors such as price², post-sale services or shorter deadlines³. Hence, any bidder can get the maximum number of points. The former contains criteria assigning points for specific technological solutions offered by a producer. Producer’s technology rarely can be updated in the short run to get more points in a given auction. Therefore, points assigned in this category are fixed and behave as if they have been assigned to particular bidders.

Bidder preference weights are constructed from the number of points assigned to particular potential bidders in criteria related to the bus and the total number of points in offer criteria. Let \bar{y} be the maximum number of points possible that could be assigned throughout criteria related to the offer and ρ_j be the number of points assigned to the bidders within bus criteria. Without loss of generality, assume $\rho_1 \geq \rho_2 \geq \dots \geq \rho_J$ and normalize the bidder

²Price is the only criterion that appears in all auctions and is always of greatest importance with a maximum of 68 points on average to be assigned. Auctions with price as the only criterion (equivalently 100 points for price) boil down to standard low-price auctions.

³If there are more offer-specific criteria besides price, the price can be easily adjusted to form a single index reflecting also the value of other offer-specific criteria.

preference weight for the most preferred bidder to one: $\theta_1 = 1$. Then:

$$\theta_j = \frac{\bar{y}}{\bar{y} - \rho_1 + \rho_j}, j \geq 2 \quad (1)$$

This formulation implies that bidder 1 wins if and only if $b_1 \leq \theta_j \cdot b_j$ for all $j \geq 2$. Therefore θ_j 's are bidder preference weights.

Appendix A describes a few frequently used scoring criteria related to bus technology and provides a simple example of an auction including construction of the scoring rule, points assignment to potential bidders and the relation between the number of points assigned in technological criteria and bidder preference weights.

Bidder preference weights θ as formulated above have interpretation in terms of the advantage of the most preferred bidder over the competitors. For example, if bidder j is assigned θ_j , it means that the most preferred bidder may submit a bid that is up to θ_j larger than j 's bids and still wins with j . We can also reformulate the weights to have an interpretation in terms of j 's position. Let:

$$\omega_j = \left(1 - \frac{1}{\theta_j}\right) \cdot 100\%$$

ω_j describes the discount in percentage points that bidder j needs to make compared to the most preferred bidder to win with them. This formulation may be more convenient in the application. The significance of technology-related criteria never exceeds 50% of the total score, which makes $\omega_j \in [0, 100]$ for all j . ω 's are more intuitive in interpretation. In the remainder of the paper, I use the term bidder preference weights to refer to ω 's, unless otherwise explained.

4 DATA

In this section, I discuss the data sources and provide a description of mechanisms governing how Polish municipal bus operators renew their fleet of buses via auctions with bidder preference weights.

4.1 SOURCES

There are two main sources of data used in this paper: auction data and fleet data.

4.1.1 AUCTION DATA

The auction data consists of the official procurement documents of 925 auctions by 189 public bus operators organized between 2006 and 2022. I have collected the documents from various sources, including a small consulting company operating in the industry; through direct requests to the operators; or scraping internet resources such as Internet archive services⁴ and document sharing platforms⁵. For each auction, I process the official documents to retrieve order characteristics, scoring rules, as well as identities, bids, and total score evaluation for participating producers.

The analysis of how the bid preference affects auction outcomes requires knowledge of bidder preference weights assigned to all the potential bidders.

Auction characteristics define a set of potential bidders, that is, a list of producers that offer products of demanded characteristics and therefore can participate in the auction. I define the set of potential bidders liberally, taking into consideration demanded length and drive of the bus and the year in which the auction is carried. The first two categories define a broad type of bus. The last accounts for producers' entry and exit. I identify potential bidders by analyzing the offer of each producer in a given year based on participation in auctions with a given set of requirements and official product brochures.

The official documents provide the total number of points assigned by the scoring rule to participating bidders. This allows me to construct bidder preference weights among participants and use them to solve for optimal bidding. However, to investigate how bidder preference weights affect the bidder's entry, I also need to know the number of points in criteria related to the bus that would be assigned to nonparticipants had they entered an auction. This requires knowledge of particular technological solutions used by producers. I learned them from a range of sources, including official product brochures, results of other auctions in which scoring criteria included a feature of interest and a given producer partici-

⁴Wayback Machine: web.archive.org

⁵docplayer.pl was particularly helpful.

pated, internet galleries, and YouTube videos, and use them to impute the number of points assigned by operators to the non-participating potential bidders.

Appendix B provides a more detailed description of data collection and processing and discusses the quality of bidder preference weights imputation for non-participating potential bidders.

4.1.2 FLEET DATA

The operators' fleets data comes from scraping a webpage <http://phototrans.eu/>, a photo gallery that has evolved into a comprehensive database about vehicles owned by operators all around the world, especially in Poland. For each bus-operator pair, dates of purchase and scrapping (or re-selling), a list of previous owners, and some limited technical details are available. The coverage of the fleet data is very good. In particular, it contains virtually all of the buses listed in auction data.

4.2 DESCRIPTIVE ANALYSIS

Table (1) describes the main features of operators' fleets and auctions they design. Operators tend to unify their fleet, that is to limit the number of distinct vehicle brands in possession. In total, operators drive buses of 60 different brands. However, an average individual fleet contains vehicles of approximately four different brands, out of which only two are still offered on the market. The distribution of brands within a fleet is not homogeneous. To investigate it, I use the Herfindahl-Hirschman Index (HHI) defined as a sum of squared shares of brands within a fleet. An average fleet HHI amounts to 0.44, suggesting presence of approximately two leading brands. To see this, compare this result with HHI for a fleet with two brands of equal share, which equals 0.5. These are likely to be the two brands that are still offered, and so can expand their fleet share in consecutive purchases. In turn, market-level HHI suggests presence of approximately five significant players.

Fleet unification is not perfect, meaning that an operator with a single-brand fleet is rarely observed. Having a single-brand fleet is nearly impossible to achieve due to a few factors. First, producers' entry and exit from the market shape brand availability and lead

to an increase in fleets’ diversity. Second, operators may buy second-hand buses from abroad, that are not sold as brand new on the market⁶. Third, producers tend to manufacture only a subset of possible bus types. Producer specialization is also an important source of fleet diversity, especially among operators with more heterogeneous needs regarding bus type.

These considerations bring reference to Harold Zurcher, a superintendent of maintenance at the Madison Metropolitan Bus Company in the seminal paper by Rust (1987). His fleet contained 162 buses of three brands. One brand (Chance) consisted of only four vehicles of special purpose, likely illustrating the impact of producer specialization. Another brand (Grumman), with 15 buses, has been introduced in the last purchase within the observation window. The remaining brand (GMC) was the main component of Zurcher’s fleet and had been the only brand delivered across 1970s.

Table 1: Operators and their auctions – summary statistics.

	count/mean	st. deviation
Operators		
# operators	189	–
fleet size	75.1	147.5
# of brands – total	60	–
# of brands – in a fleet	3.9	1.8
# of brands – in a fleet, active bidders	2.2	1.3
market fleet HHI	0.21	0.05
operator’s fleet HHI	0.44	0.26
Auctions		
# auctions	925	–
order frequency – years	1.7	2.1
order size – % of the fleet	13.5	17.0
# potential bidders	3.5	1.4
# actual bidders	1.4	0.9
Bidder Preference Weights ω		
1st most preferred bidder	0	0
2nd most preferred bidder	5.5	8.0
3rd most preferred bidder	10.3	10.5
incumbents	3.7	7.2
incumbent who won last auction	2.1	5.6
non-incumbents	9.2	11.4
participants	2.1	5.1
non-participants	10.3	11.4

⁶This used to be particularly prevalent among Polish municipal bus operators when funding sources were scarce, up until the second half of the 2000s . In the case of second-hand purchases, nearly immediate availability and low price of these buses were usually the main factors leading to purchases.

Operators tend to make frequent but small purchases. They renew 13.5% of their fleet once in 1.7 years. This reflects purchase patterns from the past as well as the current availability of funding. Neither purchasing format (auction) nor small but frequent purchases seem to be helpful tools in achieving fleet unification. Despite this fact, operators manage to keep the number of distinct brands low. Bidder preference weights are key to solving this apparent puzzle.

Operators use bidder preference weights to signal who they want to win. Bidder preference weights in table 1 are expressed in ω formulation; that is, they describe how much discount a less preferred bidder has to offer in comparison to the most preferred bidder to win with them. The weight for the most preferred bidder is by definition normalized to 0. An average bidder preference weight assigned to the second-most-preferred bidder is 5.5. That means they need to submit a bid 5.5% lower than a bid by the most preferred bidder to win with them. Interpretation of these numbers is conditional on the fact that the most preferred bidder participates in the bidding.

Operators favor producers that are already in their fleet. An average incumbent has to submit a bid lower by 3.7% than the most preferred bidder to win. Non-incumbents need on average to offer a 9.2% price discount⁷. Data reveal that an average incumbent is not the most preferred bidder. This observation may be a result of various factors. Some operators may dislike products of particular incumbents due to past bad experiences. Others may have discovered other brands that better fit their needs. It can also be optimal to maintain a roughly equal share of a few brands to insure against faulty products and be able to attract more bidders in auctions. All of these are consistent with the more preferential treatment assigned to the most recent winner (necessary discount of only 2.1%), which is likely to describe the current operator's preferences most closely. Bus operators tend to continue buying from the same sources. Similarly, Harold Zurcher's company was buying exclusively from GMC across the 1970s.

⁷In an ongoing project, I study the co-evolution of bidder preference weights and operator's fleet. I provide formal evidence on the fact that producers whose products have been introduced to the fleet enjoy more favorable treatment in consecutive auctions, whereas bidder preference weights assigned to existing incumbents remain unchanged. I also study changes in the criteria chosen by operators constructing the scoring rule in response to changes in the fleet.

Bidders' auction participation heavily depends on bidder preference weights. A typical participant is either the most preferred or nearly-most-preferred bidder. The average bid discount required to win among the entrants is 2.1%. In turn, non-participants would have needed to offer a discount of more than 10%.

Bidder preference weights affect the producer's chance of winning in two ways. They force under-preferred bidders to bid lower if they decide to bid. However, if participation in an auction is costly, the markup margin may not be large enough to accommodate an inferior bidding position resulting from bidder preference weights. This is why a typical auction could attract more than three bidders, yet two of them do not decide to submit a bid.

5 THEORETICAL MODEL

Motivated by the results obtained in the previous section, I develop a model of interactions between a single buyer (bus operator) purchasing differentiated durable goods (buses) and sellers (bus producers) selling them through auctions with bidder preference weights. The model encompasses three stages. First, the buyer chooses how much to favor or discriminate against potential bidders. Second, potential bidders decide whether to pay an entry cost and participate in the auction. Third, actual bidders—potential bidders who decided to participate in bidding and paid their entry costs, submit bids.

Assume all agents are risk-neutral. Denote the set of potential bidders by J^P . Collect indices of potential bidders who decided to enter in a set $J^A \subset J^P$.

5.1 BID PREFERENCE

I distinguish between *ex-ante* and *ex-post* bidder preference weights. Ex-ante bidder preference weights, denoted by $\tilde{\theta}_k : k \in J^P$, are chosen by the buyer in the first stage of the game and announced to potential bidders before the entry stage. They serve as a signal of how much favoritism to expect at the bidding stage. Ex-post bidder preference weights serve to rank submitted bids. They are realizations of a random variable, drawn independently across actual bidders after entry but before bidding. They follow a distribution that depends

on ex-ante bidder preference weights:

$$\theta_k \sim F_k^\theta(\cdot | \tilde{\theta}_k), k \in J^A$$

This model is a generalization of the standard approach in the literature on auctions with bidder preference weights with endogenous entry, in which θ_k is drawn from a degenerate distribution defined at $\tilde{\theta}_k$.

Ex-post bidder preference weights θ_k summarize bid credit that bidder k receives compared to their competitors. Denote the submitted bids by $b_k : k \in J^A$. The contract is granted to:

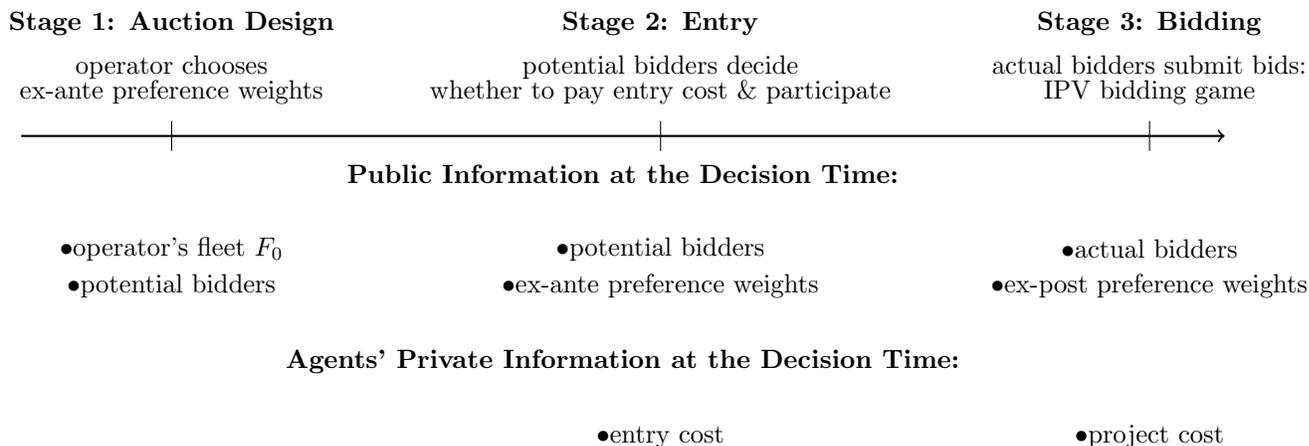
$$j = \arg \min_{k \in J^A} \theta_k b_k$$

Lower values of θ_j compared to the opponents allow actual bidder j to win despite submitting a higher bid. Note that ex-post bidder preference weights are unique up to normalization. Without loss of generality, I normalize the weight of the most preferred bidder to 1.

5.2 TIMELINE

Figure (1) summarizes timeline of the game and information available for the agents at decision time. Below I describe the three stages in reverse order and discuss modeling choices.

Figure 1: Timeline of the game.



5.3 STAGE 3: BIDDING

As a result of the entry stage, a subset of potential bidders have paid their entry costs and become actual bidders, $J^A \subset J^P$. At the beginning of the bidding stage, three pieces of information are revealed to the actual bidders. First, they learn the identities of their competitors. Second, each actual bidder draws a realization of a project completion cost:

$$c_k \sim F_k^c(\cdot) : [\underline{c}_k, \bar{c}_k] \rightarrow [0, 1], k \in J^A$$

where F_k^c is a continuous cumulative distribution function with associated density f_k^c that is strictly positive on the entire support, and such that $\frac{1-F_k^c}{f_k^c}$ is strictly decreasing. In addition, $\bigcap_{k \in J^A} [\underline{c}_k, \bar{c}_k] \neq \emptyset$ so that no bidder is sure to lose any auction. Project completion cost draws are private information of each actual bidder. Functional forms of cost distributions are known to all players. Third, a vector of ex-post bidder preference weights $\theta = \{\theta_k\}_{k \in J^A}$ is drawn and announced publicly.

A Bayesian-Nash equilibrium of the bidding game is a set of optimal bidding functions $\beta_j(\cdot)$:

$$\beta_j : [\underline{c}_j, \bar{c}_j] \rightarrow [\underline{b}_j, \bar{b}_j], j \in J^A$$

such that for each potential bidder j , $\beta_j(c_j)$ is a best response (at a project completion cost draw c_j) to actions of other players, assuming they follow their equilibrium strategies. Equilibrium depends also on the composition of actual bidders J^A and a vector of ex-post bidder preference weights θ , but since they remain constant at the bidding stage I skip them for notational convenience. I consider equilibria in which bidders bid at least their cost.

In anticipation of the fact that the unique equilibrium bidding functions $\beta_j(c)$ are strictly increasing in c , I define also an inverse bidding function $\gamma_j = \beta_j^{-1}$:

$$\gamma_j : [\underline{b}_j, \bar{b}_j] \rightarrow [\underline{c}_j, \bar{c}_j], j \in J^A$$

such that $c_j = \gamma_j(b_j)$.

Each actual bidder maximizes expected profits:

$$\begin{aligned}\pi_j(b; c_j, \theta, J^A) &= (b - c_j) \cdot \text{Prob}[j \text{ wins} | b; c_j, \theta, J^A] \\ &= (b - c_j) \cdot \prod_{\substack{k \in J^A \\ k \neq j}} \left[1 - F_k^c(\gamma_k(\frac{\theta_j}{\theta_k} b)) \right], \quad j \in J^A\end{aligned}$$

The first order conditions:

$$\gamma_j'(b) = \frac{1 - F_j^c(\gamma_j(b))}{f_j^c(\gamma_j(b))} \left[\frac{1}{|J^A| - 1} \sum_{k \in J^A} \frac{1}{b - \frac{\theta_k}{\theta_j} \gamma_k(\frac{\theta_j}{\theta_k} b)} - \frac{1}{b - \gamma_j(b)} \right], \quad j \in J^A \quad (2)$$

define a set of ordinary differential equations in the inverses of optimal bidding functions. This set together with appropriate boundary conditions characterizes equilibrium of the bidding game.

There exists a unique equilibrium of the bidding game, in which the optimal bidding functions are strictly increasing in project completion cost. The strategy of proving this claim relies on showing that each auction with bidder preference weights has a respective auction without bidder preference weights but an altered cost structure. In particular, there is a one-to-one mapping between equilibrium bidding in both games. Then we can invoke the result of [Lebrun \(2006\)](#), who proves the existence and uniqueness of equilibrium in a first-price independent values auction with general cost structure⁸⁹. I describe this idea in detail in appendix C.

⁸Classic results of [Maskin and Riley \(2000, 2003\)](#) are not sufficient in this case, since they assume common lower extremity of the support of project completion cost distributions. The mapping from auctions with bidder preference weights into alternative auctions without them shifts the support of project completion cost distributions. Therefore, even assuming common support of project completion costs across bidders would not be enough to rely on Maskin and Riley's work.

⁹An alternative way of proving existence would be to show that auctions in my setting satisfy conditions in [Reny and Zamir \(2004\)](#). However, the way of proving proposition 1 suggested in this paper provides a more intuitive way to understand the mechanisms behind existence in this particular case. By focusing on the specific problem, it also delivers characterization of the boundary conditions which is important for empirical work.

5.4 STAGE 2: ENTRY

In the second stage, J^P potential bidders decide whether to pay an entry cost and participate in the auction. They take into account a vector of ex-ante bidder preference weights $\tilde{\theta}$ chosen by the buyer at the first stage. In addition, each potential bidder draws a realization of an entry cost e_j :

$$e_j \sim F_j^e(\cdot) : \mathcal{D} \rightarrow [0, 1]$$

where F_j^e is a continuous cumulative distribution function and \mathcal{D} is a compact subset of \mathbb{R} . Potential bidders know their entry cost realization as well as distributions of their competitors' entry costs.

An equilibrium of the entry stage is a set of optimal entry rules $\delta_j : \mathcal{D} \rightarrow \{0, 1\}$ mapping entry costs into a binary decision d_j of whether or not to enter. It depends also on the set of potential bidders J^P and a vector of ex-ante bidder preference weights $\tilde{\theta}$, but since they are constant within an auction I skip them for notational convenience.

Equilibrium strategy takes the form of a cut-off rule in which the potential bidders enter if:

$$d_j = 1 \Leftrightarrow e_j \leq \pi_j$$

where $\pi_j \equiv \pi_j(\tilde{\theta}, J^P)$ denotes the expected payoff from bidding given the vector of ex-ante bidder preference weights $\tilde{\theta}$ and set of potential bidders J^P . The cut-off rule implies that the probability that j enters the auction is $p_j \equiv F_j^e(\pi_j(\tilde{\theta}, J^P))$. At the same time, the expected profit is given by:

$$\pi_j = \sum_{J^A \subset J^P} \prod_{\substack{k \in J^A \\ k \neq j}} p_k(\pi_k) \prod_{\substack{k \in J^P \setminus J^A \\ k \neq j}} (1 - p_k(\pi_k)) \int_{\theta} \int_{c_j} \pi_j(c_j, \theta, J^A) dF_j^c(c_j) dF^\theta(\theta | \tilde{\theta}) \quad (3)$$

$$\equiv \Psi_j(\pi), \quad j \in J^P \quad (4)$$

where $\pi = \{\pi_k\}_{k \in J^P}$ is a profile of expected profits from participation. Since sellers would enter only if their payoff is nonnegative, this is also a profile of entry thresholds. Therefore, the entry stage equilibrium is characterized by a fixed point equation. Given assumptions

of the model, Brouwer’s fixed point theorem guarantees the existence of equilibrium in the entry game. However, equilibrium may not be unique.

5.5 STAGE 1: AUCTION DESIGN

The buyer enters their decision stage with a current fleet of buses F_0 that reflects the history of past purchases. If a potential bidder $j = 1, \dots, N^P$ wins the auction with price b_j , the auctioneer receives a payoff $U_j(F_{1|j}, b_j)$, where $F_{1|j}$ denotes the updated fleet after j ’s win. Payoff reflects buyer’s individual preferences towards particular producers and switching costs related to introducing a new brand into the fleet. Consistently with patterns observed in the data, I allow for the possibility that an auction has no winner. In this case, the buyer receives a payoff $U_0(F_0)$. Assume u_k is a continuous function for all $k = 0, 1, \dots$

Let $b_j(\tilde{\theta})$ be the expected winning bid by potential bidder j if they decide to participate, conditional on ex-ante vector of bidder preference weights $\tilde{\theta}$. Based on the current fleet, switching costs, and their preferences, the buyer chooses a vector of an ex-ante bidder preference weights $\tilde{\theta} \equiv \{\tilde{\theta}_k\}_{k \in J^P}$ to maximize their expected utility:

$$\tilde{\theta} = \arg \max_{\tau \in \tilde{\Theta}} \sum_{k \in J^P} \text{Prob}[k \text{ wins} | \tau] \cdot U_k(F_{1|k}, b_k(\tau))$$

Given the assumption of the model, $\text{Prob}[k \text{ wins} | \tau]$ is a continuous function. Therefore $\tilde{\Theta}$ being a closed set is sufficient for existence.

5.6 DISCUSSION OVER MODELING CHOICES AND ADDITIONAL ASSUMPTIONS

5.6.1 ASSUMPTIONS ON AUCTION GAME

Entry and bidding stages build on the approach developed by [Krasnokutskaya and Seim \(2011\)](#) and [Athey, Levin, and Seira \(2011\)](#). That encompasses three major assumptions. First, the project completion cost is independent of entry cost and is revealed only for bidders who have decided to participate in the bidding and paid their entry cost, as in [Levin and Smith \(1994\)](#). This assumption is standard in the empirical literature and allows

me to separate entry and bidding. It greatly reduces the computational burden, which is particularly important in my application given arbitrary patterns of bidder preference weights.

Second, actual bidders know the identities of their opponents while submitting their bids. The market is small, with less than 10 regular bidders who repeatedly participate in auctions. It is likely that producers are well informed about each other and so can accurately predict the identities of competitors. Moreover, potential bidders often engage in a public dialog with the operator regarding order characteristics in the period between auction announcement and the bid submission deadline. It frequently concerns specific technological solutions. Therefore, even though the identity of producers asking questions is kept secret, it is straightforward to infer who expresses interest in the bidding and how likely their entry is given the operator's response to their questions.

Third, entry and project completion cost realizations are bidders' private knowledge, and are drawn independently across bidders from publicly known distributions $F_j^e(\cdot)$ and $F_j^c(\cdot)$ respectively. This assumption places my auction framework within independent private value paradigm. Entry costs are related to preparing the offer and may be a significant part of the final bids¹⁰. In my application, they refer to processing hundreds of pages of specifications, customizing products to fit the specification, making necessary arrangements with potential contractors, and properly quoting the product. Idiosyncratic variation in entry costs is attributed to differences in individual cost of labor (necessary to prepare an offer) or relation with the financial sector (proofs of financial capacity to realize the order). In turn, project completion costs concern all the costs needed to physically produce and deliver the buses. Order characteristics are specified in a detailed way, so bidders can predict their costs accurately. Existing variation in bids is therefore associated with random variation in project completion costs. This variation comes from differences in input prices faced by firms, contractual arrangements with contractors, and opportunity costs.

¹⁰Li and Zheng (2009) find that on average they amount to 8% of the winning bid in the highway moving auctions, Athey, Coey, and Levin (2013) report average entry costs to be approximately 9% of the average bid in case of timber sales.

5.6.2 MYOPIC AGENTS

Agents take myopic decisions in each auction. This assumption is equivalent to setting a discount factor in a dynamic game to zero. Although one may argue it rules out some possibly interesting inter-temporal strategies, it seems to be an accurate description of the market of city buses in Poland. Market conditions evolve in a non-stationary way, which makes it difficult for agents to form rational expectations. Considering producers who participated in at least 10 auctions, four producers exited and five producers entered throughout the sample window. Moreover, technology is developing fast. In the early 2020s, electric buses accounted for most of the sales, as they received priority in dividing government funds, and nationwide legislation required operators to increase the fraction of zero-emission buses in their fleets. This reality would sound unlikely in the early 2010s when ON-drive buses dominated the market and electric-drive buses were still in a conceptual phase of development. Uncertainty increases as new technology stimulates entry, and incumbent producers adopt it at a different pace. A non-stationary environment makes it also difficult for the producers to form rational expectations, particularly in predicting future demand and competitors.

5.6.3 RANDOMNESS IN BIDDER PREFERENCE WEIGHTS

Distinguishing between bidder preference weights chosen by the operator and used to rank bids at the bidding stage is motivated by the specific way in which bus operators choose to favor or discriminate against bidders. They assign bidder preference weights to potential bidders implicitly, through scoring rule criteria related to technological solutions. Many of these technological solutions are naturally fixed, for example, construction materials. Others may be subject to some minor innovations. For instance, a producer may introduce a new type of higher-capacity battery for electric drive. Sometimes the exact number of points a bidder would get is hard to predict. Gas consumption depends on the total weight of the bus which in turn depends on a range of additional factors including passenger information system, air conditioning system, etc. Moreover, some scoring criteria assign points to a bidder depending on the values of other offers. To account for these factors, I think of ex-

post bidder preference weights as random variables that depend on ex-ante bidder preference weights in a stochastic way.

6 ESTIMATION STRATEGY

In this section, I adapt the theoretical model to the empirical application. I outline an estimation procedure that allows me to uncover model primitives from the data.

6.1 ASSUMPTIONS OF THE EMPIRICAL MODEL

The market is populated by operators (buyers indexed by i) maintaining and renewing their fleet of buses, and producers (sellers indexed by j) selling vehicles to operators through procurement auctions with bidder preference weights.

6.1.1 AUCTION HETEROGENEITY

Auction t is characterized by contract characteristics (x_t, u_t) announced by an operator at the beginning of an auction. x_t contains a range of contract characteristics that are observed by an econometrician. It determines the set of potential bidders J_t^P containing producers who can fulfill this particular order. u_t summarizes part of contract characteristics observed by the operator and producers but unobserved by an econometrician. (x_t, u_t) affect monetary value of the order and its profitability to the bidders.

I assume that both x_t and u_t result from needs of an operator that are exogenous in my model. In particular, they are independent of the set of potential bidders J_t^P ; that is, operators do not select features of the order to exclude producers from a procurement. This assumption is consistent with the data, in which the type of buses ordered is usually related to the needs of a specific operator and remains roughly constant across auctions.

Operators announce their cost estimate b_{0t}^* after the bids are submitted and before they are opened. Bid submitted by actual bidder j is denoted by b_{jt}^* .

6.1.2 BIDDER SYMMETRY

Bidders are symmetric in terms of entry cost distributions (conditionally on a vector of observed factors facilitating entry), project completion costs, and ex-post bidder preference weights distributions:

$$F_j^e(\cdot) \equiv F^e(\cdot|Z_{ijt}), \quad F_j^c(\cdot) \equiv F^c(\cdot), \quad F_j^\theta(\cdot|\tilde{\theta}_j) \equiv F^\theta(\cdot|\tilde{\theta}_j), \quad \forall_j$$

The symmetry assumptions are motivated by the market specifics. Requirements regarding entry and bid submissions are common to all bidders and so entry costs are not likely to vary systematically among bidders, up to some observed characteristics. Project completion costs are related to the production process. Despite differences in the details of technological solutions, contemporaneous buses are fairly standardized. That includes an arrangement of important components as well as standards of equipment. Therefore, the variation in project completion costs, resulting from individual arrangements of producers and input prices they face, is not likely to be systematically different across bidders. Finally, the last equation rules out a situation in which it is easier for a particular producer to make a short time adjustment in their technology to score more points in the auction. This assumption is consistent with observed data patterns.

6.1.3 SINGLE BIDDER AUCTIONS

The majority of auctions in my data observe only one actual bidder. Single-bidder auctions pose a challenge in the low-price auction literature. Theory predicts that the single bidder bids infinity if there is no competitive pressure from other participants. This is not a feature of the data. It may be optimal for sellers to keep bids relatively low despite being a single bidder because setting unreasonably high prices may work a negative signal send to the operators on the market, discouraging them from setting up favorable bidder preference weights in auctions. A way to rationalize finite bids in single-bidder auctions is to introduce a binding reserve price. I follow the approach of [Li and Zheng \(2009\)](#) by assuming that it is a common belief among potential bidders that if they turn out to be a single bidder, they compete against the auctioneer drawing a project completion cost realization from

distribution $F^B(\cdot)$. Auctioneer’s bids are effectively a secret reserve price, that is, a reserve price that is not revealed before bidding.

A proper choice of $F^B(\cdot)$ is crucial for rationalizing single-seller bids and poses a challenge in a setting with bidder preference weights. Assuming that single bidders compete against the auctioneer who draws from the same cost distribution (as in [Li and Zheng \(2009\)](#)) would lead to a situation in which prices offered by single bidders are systematically lower than prices offered by the most-preferred bidders facing low pressure from participating but discriminated competitors. To deal with this issue, I invoke results of [Guerre, Perrigne, and Vuong \(2000\)](#) stating that the distribution of costs can be uniquely derived from distribution of bids. I assume that the auctioneer draws their costs from a distribution generated by the distribution of bids submitted by under-preferred bidders $G^B(\cdot)$. The rationale behind this modeling choice is that discriminatory bidder preference weights assigned by a buyer are an indication of the acceptable increase in price in case the most preferred bidder wins. In particular, I choose $G^B(\cdot)$ to be a distribution of bids submitted by bidders having bidder preference weights higher or equal to 5.5, which is the average bidder preference weight assigned to the second most preferred bidder in my data. This approach allows me to obtain realistic markups also in single bidder auctions¹¹.

6.1.4 UNOBSERVED HETEROGENEITY

Unobserved auction heterogeneity $u_t \leq 0$ is a uni-dimensional index distributed according to a continuous cumulative distribution function $F^u(\cdot)$ with bounded support and non-vanishing density. It is drawn independently from observed auction characteristics, the set of potential bidders, entry and project completion costs, and ex-ante and ex-post bidder preference weights. Unobserved heterogeneity allows me to explain the correlation between bids within an auction in the independent private value setting.

¹¹In the bus application, there is no directly revealed reserve price. However, approximately 7% of auctions have been canceled because the best bid in terms of the scoring rule was too high compared to the operator’s financial capability. I experimented with rationalizing bids in single-bidder auctions with a distribution of secret reserve prices estimated from data on canceled auctions. This approach failed to generate reasonable markups because the probability of being rejected for the bid being too high is relatively low in the data. The reason is that secret reserve price defined this way does not create as much pressure on a single bidder as the presence of another player would do.

6.1.5 SEPARABILITY

Project completion cost c_{jt}^* of an actual bidder j is multiplicatively separable in observed and unobserved auction heterogeneity and private information of the bidder:

$$c_{jt}^* = \exp\{\Xi(x_t)\} \cdot u_t \cdot c_{jt}$$

where $\Xi(\cdot)$ is a function known up to a set of parameters. I assume that observed heterogeneity is linear in order characteristics: $\Xi(x_t) = x_t' \xi$.

Project completion cost separability implies that optimal bidding is also multiplicatively separable (Haile, Hong, and Shum, 2003; Krasnokutskaya, 2011):

$$b_{jt}^* = \exp\{\Xi(x_t)\} \cdot u_t \cdot b_{jt}$$

where b_{jt} is the component related to strategic bidding. It is a function of a private project completion cost draw c_{jt} , hence also a random variable.

The operator's cost estimate is also separable and can be expressed as:

$$b_{0t}^* = \exp\{\Xi(x_t)\} \cdot u_t \cdot b_{0t}$$

where b_{0t} is a random term related to funds availability, drawn independently across auctions according to a distribution function $F_0(\cdot)$. b_{0t} is independent from u_t , mean independent from x_t , ex-ante and ex-post bidder preference weights, and $\mathbb{E}[\log b_{0t}] = 0$.

Multiplicative separability simplifies computation of the equilibrium bidding function across auctions. In particular, $\beta(c, u, \Xi) = \Xi \cdot u \cdot \beta(c, 1, 1)$ for any u and Ξ .

6.1.6 EX-ANTE BIDDER PREFERENCE WEIGHTS

Ex-ante bidder preference weights take three distinct values: $\tilde{\theta}_{jt} \in \{0, 1, 2\}$. $\tilde{\theta}_{jt} = 0$ denotes the most preferred bidders, who are sure to keep the most preferred bidder position in the bidding if they decide to participate. $\tilde{\theta}_{jt} = 1$ refers to bidders that are not strictly preferred,

but may expect low realizations of ex-post bidder preference weights. Eventually, $\tilde{\theta}_{jt} = 2$ is a signal that an operator is not really interested in purchasing products from j .

The intuition behind this discretization is as follows. Operators tend to specify precisely who is their producer of choice. They likely know preferred producers' technological solutions and construct scoring rule criteria accordingly. They may also decide not to use any discriminatory criteria. In both situations, the most preferred producers according to ex-ante bidder preference weight would remain the most preferred bidders according to an ex-post weight if they decide to participate in the auction. Less preferred bidders may share some solutions with the most preferred, but usually not all of them. Hence, their score would vary but may get close to the most preferred bidders depending on the construction of the scoring rule. Lastly, operators are also likely to know which technical solutions – and producers – they are not willing to accommodate, which results in very non-favorable bidder preference weights.

As a result, $F^\theta(\cdot|\tilde{\theta}_{jt} = 0)$ is a probability distribution degenerated at 0. $F^\theta(\cdot|\tilde{\theta}_{jt} = 1)$ and $F^\theta(\cdot|\tilde{\theta}_{jt} = 2)$ are continuous distributions and the latter first-order stochastically dominates the former. I parametrize the conditional distributions as Gamma distributions:

$$F^\theta(\cdot|\tilde{\theta}_{jt} = c) \sim \Gamma(\sigma_{1c}, \sigma_{2c}), \quad c \in \{1, 2\}$$

where σ_1 . and σ_2 . are shape and scale parameters respectively.

6.1.7 ENTRY PROBABILITIES

Similarly to [Athey, Levin, and Seira \(2011\)](#); [Athey, Coey, and Levin \(2013\)](#), I assume a parametric model for entry. The probability that a potential bidder will enter the auction t is defined as:

$$P[j \text{ participates in } t | z_{jt}] = \frac{\exp\{z_{jt}\eta\}}{1 + \exp\{z_{jt}\eta\}}$$

where z_{jt} is a vector of covariates and η is a vector of parameters.

Parametric specification reduces computation burden, especially in the presence of unobserved auction heterogeneity. However, it rules out potential multiplicity of equilibria. Therefore, I assume that all observations in the data are generated by the same equilibrium.

6.1.8 OPERATOR'S PAYOFFS

The operator chooses an ex-ante bidder preference weight for each potential bidder. It takes one of three possible values $\tilde{\theta}_j \in \{0, 1, 2\}$ denoting most preferred bidders, less preferred bidders, and non-preferred bidders. Let $\tilde{\theta}$ denote the vector of ex-ante bidder preference weights and $\tilde{\Theta}$ the set of possible choices. Given the relative nature of the weights, there is at least one most preferred bidder in each auction. Hence, the operator chooses a vector of ex-ante bidder preference weights from $|J^P| \cdot |J^P| \cdot 3^{|J^P|-1}$ possible choices. For example, a typical auction with three potential bidders generates $3 \cdot 3^2 = 27$ possible choices.

If potential bidder j wins auction t , the operator receives a stream of utility:

$$\zeta_b b_{jt} + \zeta'_f f_{jt}$$

where b_j is the price paid by the operator and f_{jt} contains variables describing the operator's fleet given j 's win. It may also be the case that no bidder wins the auction. In such a case, the operator receives a normalized payoff $U_0 = 0$.

The winner is determined stochastically through the auction mechanism, which depends on the vector of ex-ante bidder preference weights $\tilde{\theta}$. Operator's expected stream of utility related to updating their fleet given a choice of $\tilde{\theta} \in \tilde{\Theta}$ is:

$$\sum_{j \in J^P} \varrho_j(\tilde{\theta}) \cdot (\zeta_b \hat{b}_j(\tilde{\theta}) + \zeta'_f f_{jt}) \quad (5)$$

where $\varrho_j(\tilde{\theta})$ is the probability that potential bidder j wins given $\tilde{\theta}$, and $\hat{b}_j(\tilde{\theta})$ is the expected price submitted by j given $\tilde{\theta}$ and the fact that j wins.

Eventually, I assume that each choice of $\tilde{\theta}$ is associated with a random shock $\varepsilon_{\tilde{\theta}t}$ drawn independently and identically across alternatives and auctions from the extreme value type 1 distribution.

The total payoff from choosing $\tilde{\theta}$ is:

$$\begin{aligned}\pi_t^O(\tilde{\theta}) &= \sum_{j \in J^P} \varrho_j(\tilde{\theta}) \cdot (\zeta_b \hat{b}_j(\tilde{\theta}) + \zeta_f f_{jt}) + \varepsilon_{\tilde{\theta}t} \\ &\equiv \zeta' w_t(\tilde{\theta}) + \varepsilon_t(\tilde{\theta})\end{aligned}\tag{6}$$

where the last line uses the fact that operator's payoff is a linear function of parameters.

Specifically, $\zeta = \{\zeta_b, \zeta_f\}$ and $w_t(\tilde{\theta}) = \{\sum_{j \in J^P} \varrho_j(\tilde{\theta}) \hat{b}_j(\tilde{\theta}), \sum_{j \in J^P} \varrho_j(\tilde{\theta}) f_{jt}\}$.

Operator chooses $\tilde{\theta}$ if it delivers the highest utility:

$$\tilde{\theta} = \arg \max_{\tilde{\theta}' \in \tilde{\Theta}} \zeta' w_t(\tilde{\theta}') + \varepsilon_t(\tilde{\theta}')$$

The probability that operator i chooses $\tilde{\theta} \in \tilde{\Theta}_t$ in auction t can be written as:

$$P[\tilde{\theta}_t = \tilde{\theta} | \zeta] = \frac{\exp\{\zeta' w_t(\tilde{\theta})\}}{\sum_{\tilde{\theta}' \in \tilde{\Theta}} \exp\{\zeta' w_t(\tilde{\theta}')\}}$$

6.2 ESTIMATION STRATEGY

Empirical model assumptions lead to the following list of primitives to be estimated:

- parameters of observed auction level heterogeneity ξ
- distribution of unobserved auction level heterogeneity $F^u(\cdot)$
- distribution of project completion costs $F^c(\cdot)$
- parameters σ of conditional distribution of ex-post bidder preference weights $F^\theta(\cdot | \tilde{\theta})$
- parameters η of entry probabilities
- parameters ζ of operators' payoff

Observed and unobserved auction heterogeneity as well as project completion cost are primitives of the bidding stage. Conditional distributions of ex-ante bidder preference weights and participation probabilities are primitives of the entry stage and allow me to recover entry costs. The last part refers to stage 1, the operator's problem. The remainder of this section describes how each of the primitives is estimated from the data.

6.2.1 OBSERVED HETEROGENEITY

In the first step, I separate observed auction level heterogeneity from the bids by applying bid homogenization (Haile, Hong, and Shum, 2003). Observed heterogeneity is linear in order characteristics: $\Xi(x_t) = x_t'\xi$. The multiplicative structure of bids allows me to write them as:

$$\log b_{jt}^* = x_t'\xi + \log u_t + \log b_{jt}, \quad j = 0, 1, \dots \quad (7)$$

where ξ is a vector of parameters.

x_t includes year, drive, length, size of the order, leasing indicators, additional items required in the order (e.g., electric battery chargers), delivery deadline, and length of warranties required. In addition, operators tend to have their standards regarding additional equipment, like types of driver's compartment, passenger information systems or air conditioning. I include operator's fixed effects for operators with a total of 10 or more observations available. I add operator size to the specification as a proxy for financial capabilities and an indicator for European Funds. The remaining term u_t summarizes factors unaccounted for at the auction level, including deviations from the operator's standard requirements, or prestige gains to the producer of selling to large and recognized operators.

I estimate the parameters of the equation (7) taking into account potential endogeneity resulting from the existence of unobserved fixed effect $\log u_t$ at the auction level. The estimation approach is described in detail in appendix E.

6.2.2 UNOBSERVED HETEROGENEITY & PROJECT COMPLETION COSTS

I apply non-parametric deconvolution methods (Kotlarski, 1966; Li and Vuong, 1998; Krasnokutskaya, 2011) to homogenized bids $bu = \log b_{jt}^* - x_t'\xi$ to estimate distributions of unobserved heterogeneity $F^u(\cdot)$ and individual project completion costs $F^c(\cdot)$. This approach relieves me of necessity of imposing functional form assumptions on these distributions. It also provides estimates for bounds of the support of obtained distributions, allowing me to closely follow the theoretical model which assumes bounded support for the individual project completion cost components.

The estimation proceeds in two steps. First, I estimate the distributions of unobserved heterogeneity $F^u(\cdot)$, strategic components of bids in single bidder auctions $G^1(\cdot)$, and strategic bid components submitted by bidders whose bidder preference weight is above 5.5 $G^B(\cdot)$. The latter serves as an approximation of auctioneer strategies rationalizing bidding in single-bidder auctions. In the second step, I follow insight of [Guerre, Perrigne, and Vuong \(2000\)](#) to derive the distribution of individual project completion costs using estimated strategic bid distributions. I focus exclusively on single bidder auctions, as in this case the bid distribution is not affected by participation patterns and bidder preference weights. Further explanations and technical details regarding the estimation method in this subsection can be found in appendix [F](#).

6.2.3 EX-ANTE BIDDER PREFERENCE WEIGHTS

The most preferred bidders in terms of ex-ante bidder preference weights are sure to enjoy the most preferred position also at the bidding. Hence, whenever ex-post bidder preference weight indicates the most preferred position $\theta_{jt} = 0$, it follows that the ex-ante bidder preference weight does the same: $\tilde{\theta}_{jt} = 0$. If $\theta_{jt} > 0$, it could be that $\tilde{\theta}_{jt} = 1$ or $\tilde{\theta}_{jt} = 2$. Therefore, observations of θ_{jt} are random draws from a mixture of distributions with two components $F^\theta(\cdot|\tilde{\theta}_{jt} = 1)$ and $F^\theta(\cdot|\tilde{\theta}_{jt} = 2)$ and mixing weights $\text{Prob}[\tilde{\theta}_{jt} = 1|z_{jt}^\theta]$ and $\text{Prob}[\tilde{\theta}_{jt} = 2|z_{jt}^\theta]$ respectively. Mixing weights denote probability of observing θ_{jt} given that it has been drawn from $F^\theta(\cdot|\tilde{\theta}_{jt} = 1)$ and $F^\theta(\cdot|\tilde{\theta}_{jt} = 2)$ respectively. They may depend on a vector of exogenous covariates z_{jt}^θ .

Since the draws of ex-post bidder preference weights are mutually independent, the problem boils down to a standard latent class model with two latent classes. Observation jt 's contribution to the likelihood can be expressed as:

$$\ell(\theta_{jt}) = \sum_{c=1}^2 \text{Prob}[\tilde{\theta}_{jt} = c | z_{jt}^\theta] \cdot f_j^\theta(\theta_{jt} | \tilde{\theta}_{jt} = c)$$

By assumption, $f_j^\theta(\theta_{jt}|\tilde{\theta}_{jt} = c)$ are densities of Gamma distribution parametrized by $(\sigma_{1c}, \sigma_{2c})$ for $c \in \{1, 2\}$. I also assume that the mixing probabilities can be written as:

$$\text{Prob}[\tilde{\theta}_{jt} = 1|z_{jt}^\theta] = \frac{\exp\{z_{jt}^\theta\sigma_3\}}{1 + \exp\{z_{jt}^\theta\sigma_3\}}, \quad \text{Prob}[\tilde{\theta}_{jt} = 2|z_{jt}^\theta] = \frac{1}{1 + \exp\{z_{jt}^\theta\sigma_3\}}$$

where σ_3 is a vector of parameters. I estimate $\sigma = \{\sigma_1, \sigma_2, \sigma_3\}$ using maximum likelihood methods.

z_{jt}^θ 's are not necessary to identify the conditional distributions of ex-post bidder preference weights, but help to predict which observations in the data come from which conditional distribution. In my application, z_{jt}^θ 's contain a series of variables describing j 's contribution to the operator's fleet, including indicators of being in the fleet, past wins, past purchases as second-hand vehicles, and overall discrimination potential of an auction. I impute the ex-ante bidder preference weights using estimated mixing weights:

$$\tilde{\theta}_{jt} = \arg \max_{c \in \{1,2\}} \text{Prob}[\tilde{\theta}_{jt} = c|z_{jt}^\theta]$$

Without additional variation in z_{jt}^θ , the estimated mixing probabilities would be constant across the sample, making it difficult to impute ex-ante bidder preference weights.

6.2.4 PARTICIPATION PROBABILITIES

Participation probabilities are expressed as:

$$P[j \text{ participates in } t|z_{jt}; \zeta] \equiv P[d_{jt} = 1|z_{jt}, u_t; \zeta] = \frac{\exp\{z_{jt}\zeta\}}{1 + \exp\{z_{jt}\zeta\}}$$

where z_{jt} is a vector of covariates, ζ is a vector of parameters. We expect that auction profitability as expressed by auction level heterogeneity $\Xi(x_t) \cdot u_t$ may affect entry, hence it is a part of z_{jt} . The auction heterogeneity term creates a challenge in estimation, as it is observed by potential bidders deciding whether to participate in an auction, but is typically unobserved by an econometrician.

Fortunately, the data contains useful information regarding u_t . Its distribution is recovered in one of the previous steps. Moreover, homogenized bids contain repeated (but noisy)

measurements on realizations of the unobserved auction heterogeneity. Taking advantage of the fact that the support of all the components of homogenized bids is bounded and bounds have already been estimated, I derive bounds for each realization of u_t in the sample. Let $\{\bar{u}, \underline{u}\}$, $\{\bar{b}, \underline{b}\}$ and $\{\bar{b}_0, \underline{b}_0\}$ denote bounds of the support of unobserved heterogeneity u , strategic bid components b and operator's price estimate b_0 . For each submitted homogenized bid bu_{jt} we can retrieve bounds for the realization of u_t :

$$u_t \in [\max\{bu_{jt} - \bar{b}, \underline{u}\}, \min\{bu_{jt} - \underline{b}, \bar{u}\}], \quad j \in J_t^A$$

Analogously, for the homogenized operator's cost estimate it follows that:

$$u_t \in [\max\{bu_{0t} - \bar{b}_0, \underline{u}\}, \min\{bu_{0t} - \underline{b}_0, \bar{u}\}]$$

Intersecting all these intervals for repeated homogenized measurements within an auction returns the interval in which unknown realization of u_t falls in. Denote it by $[\underline{u}_t, \bar{u}_t]$. Using this information, for each auction t I consider the conditional distributions of u , given the $u \in [\underline{u}_t, \bar{u}_t]$:

$$F^u(u | u \in [\underline{u}_t, \bar{u}_t]) = \frac{F^u(u) - F^u(\underline{u}_t)}{F^u(\bar{u}_t) - F^u(\underline{u}_t)} \equiv F_t^u(u)$$

with associated density $f_t^u(u) = \frac{\partial F_t^u(u)}{\partial u}$.

Integrating over unobserved heterogeneity and making use of the fact that entry costs are independent across auctions and bidders, the log-likelihood function can be expressed as:

$$\begin{aligned} \ell(\zeta) = & \sum_t \sum_j \mathbb{1}[d_{jt} = 1] \log \left(\int_u \mathbb{P}[d_{jt} = 1 | z_{jt}, u_t; \zeta] dF_t^u(u) \right) \\ & + \mathbb{1}[d_{jt} = 0] \log \left(\int_u (1 - \mathbb{P}[d_{jt} = 1 | z_{jt}, u_t; \zeta]) dF_t^u(u) \right) \end{aligned}$$

which is maximized over ζ to obtain the desired parameters. The covariates z_{jt} include variables relevant for entry decision, including the observed auction heterogeneity Ξ_t , own ex-ante bidder preference weight, and ex-ante bidder preference weights of other potential bidders. I also include a dummy for incumbent status to investigate potential incumbent advantage.

6.2.5 ENTRY COSTS

Entry is characterized by a threshold rule. A potential bidder decides to participate in an auction if the expected payoff from participation given a vector of ex-ante bidder preference weights $\tilde{\theta}$ and a set of potential bidders J^P , $\pi_j(\tilde{\theta}, J^P)$, exceeds a realization of entry costs e_j :

$$d_{jt} = 1 \Leftrightarrow e_{jt} \leq \pi_j(\tilde{\theta}, J^P, u_t)$$

Given the reduced form model of participation probabilities, I cannot recover the distribution of entry costs directly. However, the threshold rule is informative about bounds on realizations of entry costs. Specifically, the expected profits of potential bidders who did not enter must lie below their realization of e_j . Conversely, the expected profits of participants exceed their entry cost draw.

Obtaining expected profits $\pi_j(\tilde{\theta}, J^P, u_t)$ is a numerically challenging task, as they integrate out within-auction profits over a project completion cost draw c_j , vector of ex-post bidder preference weights θ conditionally on a vector of ex-ante bidder preference weights $\tilde{\theta}$, and the set of actual bidders J^A conditionally the set of potential bidders J^P :

$$\pi_j(\tilde{\theta}, J^P) = \sum_{\substack{J^A \subset J^P \\ j \in J^A}} \prod_{\substack{k \neq j \\ k \in J^A}} P[d_{kt} = 1 | \tilde{\theta}, J^P] \prod_{\substack{\ell \neq j \\ \ell \in J^P \setminus J^A}} (1 - P[d_{\ell t} = 1 | \tilde{\theta}, J^P]) \int_{\theta} \int_c \pi_j(c; \theta, J^A) dF^c(c) dF^{\theta}(\theta | \tilde{\theta})$$

Equilibrium bidding functions do not have a closed form and need to be solved numerically. To do so, I modify shooting algorithms for solving two-point boundary problems to the setting with arbitrary ex-post bidder preference weights. The main challenge lies in accommodating bid bifurcation ([Hubbard and Kirkegaard, 2019](#)) resulting from potentially different supports of equilibrium bids among bidders [Lebrun \(2006\)](#). I describe this in detail in [appendix D](#).

Integration over the conditional distributions of ex-ante bidder preference weight is multi-dimensional. To maximize the accuracy of numerical integration and limit the number of necessary function evaluations, I use generalized Laguerre quadrature which fits particularly well to the framework with F^{θ} parametrized as Gamma.

6.2.6 OPERATOR’S PROBLEM

The definition of operator’s problem leads to a multinomial choice framework. This case is special compared to other models in the literature because the operator essentially chooses a distribution of winning probabilities and expected prices over a range of potential bidders instead of choosing a product itself. I follow the strategy of [Petrin \(2002\)](#) and [Gentzkow \(2007\)](#) to use a rich specification that would free the model from heavy dependence on the *logit* idiosyncratic taste shock associated with choices of $\tilde{\theta}$.

In the baseline specification, I assume that operator’s stream of utility associated with the win of producer j depends on a dummy indicating whether j ’s products are already in the fleet, a dummy of whether j has won an auction within the past 3 years, a dummy of whether j ’s buses have arrived as second-hand buses within last three years, indicator of the fact that the current generation of j ’s products is already in the fleet, indicator of whether j ’s buses of the same drive are already in the fleet and producer’s share in the fleet given their win. I also include producers’ fixed effects, which I model as random coefficients to account for unobserved qualities of a match in operator-producer pairs ij . The specification of random coefficients is standard: I assume they follow a normal distribution with mean and variance to be estimated. I follow the simulated maximum likelihood strategy, averaging over draws of random tastes and unobserved auction-level heterogeneity.

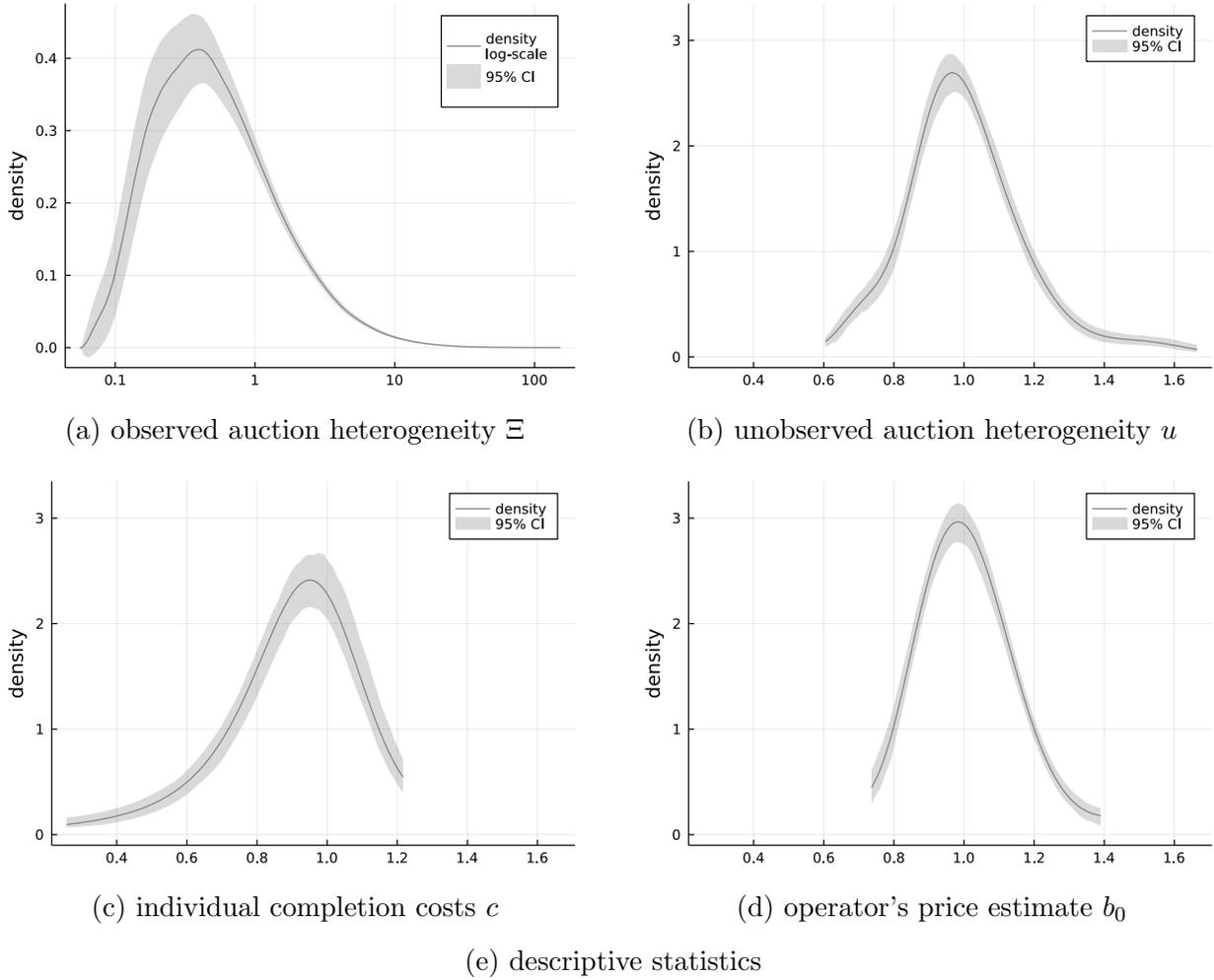
7 ESTIMATION RESULTS

This section presents the results of the estimation. I divide it into three subsections, discussing the bidding, entry, and operator stages respectively. Unless stated differently, all monetary terms are expressed in millions of 2010 USD.

7.1 BIDDING STAGE

Project completion costs are a product of observed (Ξ) and unobserved (u) auction heterogeneity and individual component c . [Figure \(2\)](#) presents the estimated densities of these components.

Figure 2: Estimated densities of project completion cost components.



	min	med	max	mean	st dev
observed heterogeneity (millions 2010 USD)	0.056	1.948	153.32	5.029	9.082
unobserved heterogeneity	0.604	0.99	1.665	1.008	0.179
price estimate	0.736	1.0	1.39	1.008	0.131
completion costs	0.257	0.916	1.217	0.89	0.185

The distribution of observed auction heterogeneity is skewed with a long right tail, as depicted in panel (2a). Its variance is large compared to mean (panel 2e). These observations suggest that the auction orders reflect diverse needs of the operators. The market observes a few outstandingly large contracts. The distribution of unobserved auction heterogeneity (panel 2b) is more symmetric than the observed heterogeneity component, and also much more concentrated around the mean. However, the longer right tail indicates presence of infrequent auctions with high realizations of u .

Since not only costs but also optimal bidding is separable in observed and unobserved auction level heterogeneity, these two components describe auction level profitability. Since the observed heterogeneity realization is technically a fitted value in a regression in which the explanatory variable is price, I think of it in terms of the monetary value of the project. The unobserved heterogeneity shifts it up or down due to factors hidden for the econometrician, including specification details not included in the sample or less tangible factors such as prestige of realizing contracts for big operators.

The lower bound of the individual component is nearly three times smaller than the lower bound of the distribution of operator’s price estimates (panel 2d). The latter is a good proxy of a variation in submitted bids, which is free from the effects of bidder preference weights and participation patterns. Bidders’ opportunity for large mark-ups is concentrated only in the low-density regions of the left tail (panel 2c). The difference in median bidders’ individual cost components and operator’s price estimates amounts to approximately 10% (panel 2e) and remains comparable at the right tail.

7.2 ENTRY STAGE

7.2.1 EX-ANTE BIDDER PREFERENCE WEIGHTS

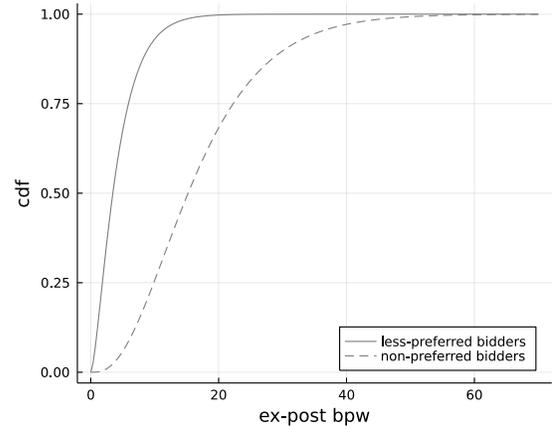
Figure (3) presents results of the estimation of ex-ante bidder preference weights distributions and associated latent class mixing weights. The upper part of panel (3a) shows the summary statistics of conditional distributions of ex-post bidder preference weights given ex-ante preferential status. The most preferred bidders are sure to keep this status if they decide to participate, regardless of the entry decisions of other bidders. The less preferred bidders receive ex-post bidder preference weight on average of 4.31. This number is comparable to the bidder preference weight of the second most preferred bidder observed in the raw data. Less-preferred bidders have a good chance of receiving favorable weights. Non-preferred bidders also have a chance for small draws, but it is slight. On average, they are assigned ex-post bidder preference weight exceeding 17.

These results should be interpreted bearing in mind that final preferential treatment depends on entry. If the most preferred bidder enters, they retain the most preferred status,

Figure 3: Conditional distributions of ex-post bidder preference weights: $F^{\theta}(\cdot|\tilde{\theta})$.

latent class distributions			
bidders	mean	st. dev.	N
most-preferred ($\tilde{\theta} = 0$)	0	0	938
less-preferred ($\tilde{\theta} = 1$)	4.31	3.42	359
non-preferred ($\tilde{\theta} = 2$)	17.06	9.81	602

(a) parameter estimates



(b) cumulative distribution functions

or zero ex-post bidder preference weight. However, the preferential treatment of actual bidders that have been assigned a positive ex-ante bidder preference weight depends on the configuration of other participants and their draws of ex-post bidder preference weights.

I use estimates of the mixing weights associated with latent distribution to infer ex-ante bidder preference weights. The mixing weights describe the probability of being assigned $\tilde{\theta} = 1$ as opposed to $\tilde{\theta} = 2$, given a set of covariates. I explain this probability using a range of variables related to the operator’s fleet, and operator and order characteristics. The estimates are attached in table (11) in appendix G. The number of most-preferred bidders is nearly equal to the number of auctions, confirming substantial degree of favoritism.

Panel (3b) shows that the conditional distribution of ex-post bidder preference weights among non-preferred bidders first-order stochastically dominates the analogous distribution among less-preferred bidders. This feature has not been imposed on the estimation routine. Estimates recover assumed patterns, strengthening the distinction between less- and non-preferred potential bidders. That speaks in favor of latent class specification and estimation reliability.

7.2.2 ENTRY PROBABILITIES

Table (2) presents estimates of the auction participation probability. To make the results interpretable, I focus on the average marginal effects (continuous variables) and average marginal changes (discrete variables) of explanatory variables on participation probability.

Table 2: Entry stage estimates – average effects on participation probability.

	average effects
competitive environment	
own weight: $\tilde{\theta} = 1$	-0.159*** (0.003)
own weight: $\tilde{\theta} = 2$	-0.246*** (0.015)
competitors' weights: 1st best $\tilde{\theta} = 1$	0.115*** (0.003)
competitors' weights: 1st best $\tilde{\theta} = 2$	0.294*** (0.003)
competitors' weights: 2nd best $\tilde{\theta} = 1$	0.023*** (0.004)
competitors' weights: 2nd best $\tilde{\theta} = 2$	-0.019*** (0.005)
competitors' weights: 3rd best $\tilde{\theta} = 1$	0.088*** (0.002)
competitors' weights: 3rd best $\tilde{\theta} = 2$	0.047*** (0.008)
# potential bidders	-0.035 (0.092)
fleet composition	
incumbent	-0.003 (0.005)
won within past 3 years	0.182*** (0.007)
2nd hand delivery within past 3 years	0.004* (0.002)
brand's current generation in the fleet	0.196*** (0.006)
producer's bus of the same drive in the fleet	-0.008 (0.004)
compatibility index (non-incumbents)	-0.01 (0.009)
# brand in the fleet	-0.014 (0.115)
characteristics of the order	
ordered drive already in the fleet	-0.095*** (0.011)
auction profitability $\Xi(x_t) \cdot u_t$	0.032 (1.089)
N	3214

Delta method standard errors in parentheses. p. val: *** ≤ 0.001 , ** ≤ 0.005 , * ≤ 0.01 . Average marginal effects are obtained as the average of marginal effects for continuous variables, expressed in standard deviation units or marginal change effects for categorical variables. The index of compatibility between a non-incumbent producer and the operator's fleet is constructed as follows. First, I calculate correlation coefficients between the number of buses in operators' fleets by all pairs of producers and average it over time. The compatibility index for a non-incumbent is an average of the correlations between the non-incumbent and incumbents, weighted by the fleet share of the incumbents. The compatibility index is by definition normalized to the interval $[-1, 1]$. High values of the index indicate high levels of compatibility between non-incumbent and the operator's fleet.

I distinguish three main channels affecting bidders' decision to enter: factors related to the auction competitive environment set by the operator, factors related to the operator's fleet and factors related to characteristics of the order.

Potential bidders take into consideration both their own and competitors' ex-ante bidder preference weights. Potential bidders who are less-preferred ($\tilde{\theta} = 1$) participate in an auction with a probability 15.9 percentage points smaller than their most preferred competitors. This effect strengthens among the non-preferred potential bidders, whose entry rate is smaller by 24.6 percentage points. Ex-ante bidder preference weights assigned to the competitors are also important. Intuitively, less preferable treatment of other potential bidders decreases competitive pressure and increases the chances of winning at the bidding stage. As a result, potential bidders whose most preferred opponent received $\tilde{\theta} = 1$ participate with 11.5 percentage points higher probability than if facing opponents with $\tilde{\theta} = 0$. Those who are to compete against solely non-preferred bidders participate with a probability larger by 29.4 percentage points. Conditional on own and competitors' ex-ante bidder preference weights, the number of potential bidders does not play a big role in deciding whether to participate in the auction.

Conditional on the competitive environment, the operator's fleet structure explains significantly potential bidders' auction participation patterns. This can be interpreted as evidence of incumbent advantage among potential bidders. Incumbent advantage refers to a situation in which it is *easier* for incumbent potential bidders to participate in an auction. That may be related to the benefits of an established connection with the operator. With buses already in the operator's fleet, producers frequently have set up a network of authorized workshops in the proximity of the operator's depot as well as spare parts delivery chains. These factors are often required in auction specifications and contribute to entry costs. Additionally, since operators tend to keep their standards roughly fixed across auctions, previous deliveries ensure that producers have already implemented these standards in their production lines.

Even though the incumbency status itself does not affect entry, producers who have delivered their buses recently are more likely to enter—by up to 18.2 percentage points if new buses have been delivered within the past three years. This finding supports the incumbent advantage hypothesis, as recent orders tend to be most correlated with the current one.

In the same spirit, auctions by operators who possess the newest generation of producer’s products in their fleets are more likely to attract these producers. Incumbent advantage strengthens the lock-in between operators and producers.

Eventually, auction characteristics also affect entry. If the operator orders buses with a type of drive that has not been previously exploited, they may expect increased entry. Consistently with intuition, more profitable auctions attract more bidders, however, the estimate lacks precision.

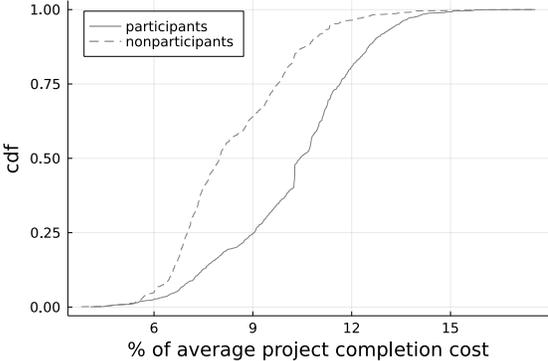
7.2.3 ENTRY COSTS

The comparison between average expected profits among potential bidders who did not enter and the participating bidders reveals bounds on average entry costs. Figure (4) presents the results.

Figure 4: Expected profits from the participation in auction.

	participation	
	no	yes
% of average project completion cost	8.41	10.27
USD (millions, 2010)	0.32	0.53

(a) bounds on average entry costs



(b) cumulative distribution functions

Panel (4a) indicates that the average entry cost lies between 8.41% and 10.27% of average project completion costs. These numbers are high, but comparable with entry costs obtained in the literature (Li and Zheng, 2009; Athey, Levin, and Seira, 2011). Estimated bounds are relatively tight. The difference between them amounts to approximately 1.86% of the average project completion cost.

Panel (4b) presents the empirical cumulative distribution of expected profits from participation among participants and non-participants. The cumulative distribution function

of entry costs lies between these two lines. In particular, it suggests that entry costs are contained in an interval between 5% and 15% of average project completion costs.

7.3 OPERATOR'S STAGE

Table (3) presents estimates of the operator's utility parameters. Consistent with descriptive evidence, operators favor incumbent producers. The preference towards winners of more recent auctions is even stronger. Interestingly, a recent delivery of second-hand buses does not significantly improve producers' chances of being favored. This finding suggests that second-hand purchases are driven by other factors than purchases of new buses, for example, availability. The lock-in type of relationship between operators and producers goes beyond solely technical aspects of the buses, as indicated by a non-significant estimate at the dummy indicating that the current producer's generation of products is already in the operator's fleet. Not only do operators derive utility from overall fleet unification, but they also prefer the unification of sub-fleets defined by bus drives. Producer's share in the fleet conditional on their win increases the operator's utility, suggesting the existence of costs related to maintaining more diverse fleets. However, the coefficient is not precisely estimated.

The estimated coefficients are expressed in units of utility and so cannot be directly interpreted. To be able to say more about the degree of switching costs, I first express them in monetary terms by dividing them by the estimate of the price coefficient. The resulting numbers are known as Willingness-To-Pay (WTP) in the discrete choice literature. Second, I divide them by the average value of the order as measured by auction level heterogeneity $\Xi(x_t) \cdot u_t$. Table (4) presents the results of a subset of variables.

On average, incumbent's win is priced at 10.58% of the order value, holding other factors constant. Operators are willing to surrender an additional 9.61% of the order value to ensure unification within the bus drive. WTP for winning of a producer that won a recent auction amounts to 7.99% of the order value. That means operators are willing to pay on average up to 28.18% of the order value for the win of the most suitable incumbent. The WTP for an incumbent win in terms of value of the order is very high, which partially results from the tendency to carry small and frequent orders.

Table 3: Estimates of operator’s utility parameters.

price	
expected price	-2.106*** (0.364)
fleet	
brand in the fleet	3.506** (1.097)
won within past 3 years	3.184*** (0.754)
2nd hand delivery within past 3 years	0.816 (0.794)
brand’s current generation in the fleet	0.682 (0.88)
producer’s bus of the same drive in the fleet	2.649* (1.09)
producer’s share if fleet if wins	3.032* (1.518)
brands	
dummies	✓
random effects	✓
N buyers	176
N auctions	926
N alternatives	59428

Standard errors in parentheses. p. val: *** ≤ 0.001 , ** ≤ 0.005 , * ≤ 0.01 . The specification with random coefficients will be added in the next draft. Current estimates of standard errors do not account for uncertainty related to using estimates obtained in the previous steps.

Table 4: Quantification of disruption costs: Willingness-To-Pay approach.

brand in the fleet	10.58** (3.84)
won within past 3 years	9.61*** (2.87)
producer’s bus of the same drive in the fleet	7.99* (3.5)

Average effects expressed in % of the order value $\Xi(x_t) \cdot u_t$. Delta method standard errors in parentheses. p. val: *** ≤ 0.001 , ** ≤ 0.005 , * ≤ 0.01 .

8 COUNTERFACTUAL ANALYSIS

In this section, I use estimates of the structural model to study counterfactual scenarios. The main motivation is to find ways to increase buyers’ welfare. I show that bid preference programs can balance the trade-off if an auction attracts sufficiently many bidders, whereas forcibly promoting competition while ignoring the underlying lock-in relationship between buyers and incumbent sellers would lead to counter-productive outcomes.

8.1 SEPARATING THE EFFECTS OF DISCRIMINATION AND AUCTION PARTICIPATION

Will operators face lower prices if they set less discriminatory bidder preference weights holding participation fixed, or if they manage to attract more bidders holding discrimination fixed? In this section, I separate the effects of bidder preference weights and participation on prices using the bidding stage estimates. I focus on ex-post bidder preference weights to keep the analysis closer to the raw data evidence.

I start with an investigation concerning how the ex-post bidder preference weights affect producers' bidding. I consider a counterfactual scenario in which I increase the bidder's preference weight by a standard deviation for each auction and each actual bidder. I study how this affects the average bid of a bidder whose weight increased, average bids of their competitors, and the average winning bid. In this exercise, I keep the participation fixed, considering solely participation patterns from auctions in my data. For meaningful analysis, I restrict my attention to auctions with at least two actual bidders. The results are summarized in table (5)

Table 5: Effects of discrimination on average bidding.

	mean	st dev
affected bidder's bids	-6.339	0.318
others bidders' bids	1.5	0.738
winning bids	0.248	0.824

Change in average optimal bidding between data and counterfactual scenario as % of data values.

Considering a set of auctions in my sample, increasing bidder preference weight for one of the bidders affects mainly their bidding behavior. The affected bidder decreases their bids by 6.3% on average. The competitors do not react strongly, increasing their bids only by 1.5%. The expected winning bid increases only by 0.25%. This is because the potential discounts resulting from the decrease in price by the affected bidder are annihilated by their decreased probability of winning. Holding participation fixed, the discrimination channel does not affect significantly the costs of procurement but shifts contract allocation towards more preferred bidders.

In the second step, I switch my attention to the effects of participation. I consider a counterfactual scenario in which I assume an additional potential bidder decided to enter. I study how this affects average prices of original participants and average winning bids. To keep the analysis more realistic, I assume that the additional entrant is the non-participant with the lowest (the most favorable) ex-post bidder preference weights in the data. Table (6) presents the result of this counterfactual exercise.

Table 6: Effects of participation on average bidding.

	mean	st dev
true participants' bids	-0.499	7.333
winning bids	-5.09	3.615

Change in average optimal bidding between data and counterfactual scenario as % of data values.

Given the sample, the presence of an additional actual bidder leads to a decrease in winning bids by 5.1%. The average effect of the additional participant on the bids of true participants is small. However, its large variance suggests heterogeneous impacts. This is consistent with intuition; as in the counterfactuals scenario, the additional participant's weight may have been much larger or much smaller compared to the true participants.

Results of this counterfactual exercise suggest that the bid preference program itself does not have to be a source of a significant increase in the expected winning price. It rather reallocates the winning probabilities among bidders. In turn, participation is the leading margin of potential reductions in procurement costs. This indicates that if the auctioneer is able to attract a sufficient number of bidders, bid preference programs may be suitable for balancing the trade-off between price and disruption costs. Specifically, preferred bidders are more likely to win whereas the competitive pressure from other auction participants would keep the prices low.

8.2 THE VALUE OF BID PREFERENCE

In this section, I use the full structural model to perform counterfactual exercises. I show that forcibly promoting competition while ignoring disruption costs may lead to counterproductive results. In turn, bid preference programs may allow operators' to balance their

trade-off between achieving low prices and avoiding switching costs if buyers can ensure sufficient participation. The computational approach behind the counterfactual exercises is attached in appendix [H](#).

The baseline scenario assumes the original data environment, that is, auctions with bid preference and estimated probabilities of entry. I compare the outcomes under the baseline scenario to outcomes generated in three counterfactual scenarios. First, I take away the possibility of favoring bidders by assuming that each auction is a low-price auction, in which the cheapest bid wins. Comparing buyers' welfare attained in the low-price auction and the baseline scenarios describes the value of bid preference to the operators. Second, I maintain the possibility of using bid preference but assume all potential bidders participate in each auction. The goal of analyzing the perfect participation scenario is to verify whether the bid preference help buyers balance their trade-off by allowing them to reallocate winning probabilities towards incumbent bidders while managing to keep the prices low. Third, I consider a combination of the two mentioned scenarios, that is low price auctions with full participation.

I use compensating variation to study changes in operators' welfare. Specifically, I ask how much money operators need to be given (or taken) to be indifferent between the baseline and counterfactual scenarios. To answer this question, I obtain the expected buyers' utility under the baseline scenario EU_0 and analogous expected utility under counterfactual scenarios EU_i , $i \in \{L, S, B\}$ where L stands for low price auctions, S for full participation, and B for both. The resulting compensating variation is given as:

$$CV_i = \frac{EU_i - EU_0}{\zeta_b}, i \in \{L, S, B\}$$

where ζ_b is the coefficient at price in the operator's utility function.

Table [7](#) presents the results. The first row describes the calculated compensating variation. Switching an auction format to low-price auctions has a detrimental effect on operators' welfare. On average, operators are willing to pay 0.56 million 2010 USD for being able to use bid preference in an auction. In only 3.6% auctions the low-price setting would actually improve buyer's welfare. In turn, the operators are willing to pay 0.2 million to be able to

Table 7: Counterfactual exercise—compensating variation and contributing factors.

		Scenario		
		low price auctions	perfect participation	both
compensating variation	mean	-0.56	0.2	-0.37
	% positive	3.63	52.76	14.62
Δ price	mean	-0.03	-0.37	-0.3
Δ # participants	mean	0.17	1.88	1.89
Δ # brands if fleet*	mean	0.35	0.43	0.62

Compensating variation and prices expressed in millions of 2010 USD.

attract all potential bidders, while being able to favor some of them. Increasing participation while still allowing for auction favoritism improves welfare in more than half of the auctions. Switching to a low-price auction format and at the same time ensuring full participation has a negative impact on buyers’ welfare.

Switching to a low-price auction format leads to a small decrease in prices (30k dollars per auction). This is driven by the insufficient increase in auction participation—on average auctions under the low-price scenario observe only .17 actual bidders per auction more than in the baseline scenario. Removing favoritism leads to a relative increase in entry by non-incumbents. After 4 auctions, an average fleet under the low-price scenario contains 0.35 more brands.

Perfect participation by construction tackles the problem of insufficient entry. The decrease in expected prices is a magnitude larger than under low-price auctions, and in over 52% of cases outweighs the increase in disruption costs. This is possible because the bid preference program allows the operators to re-allocate winning probability towards the incumbents despite increased entry. Without bid preference, the decrease in price associated with increased participation does not offset the increase in switching cost, as depicted in the last column of table 7.

The effects of switching to a low-price auction format and ensuring full participation are heterogeneous across auctions. Figure 5 shows the calculated compensating variation by the size of the order. In both scenarios, compensating variation is increasing in the number of buses ordered in an auction. Intuitively, the significance of disruption costs decreases relative to the value of the order, being replaced by price as the main contributor to welfare. Low-

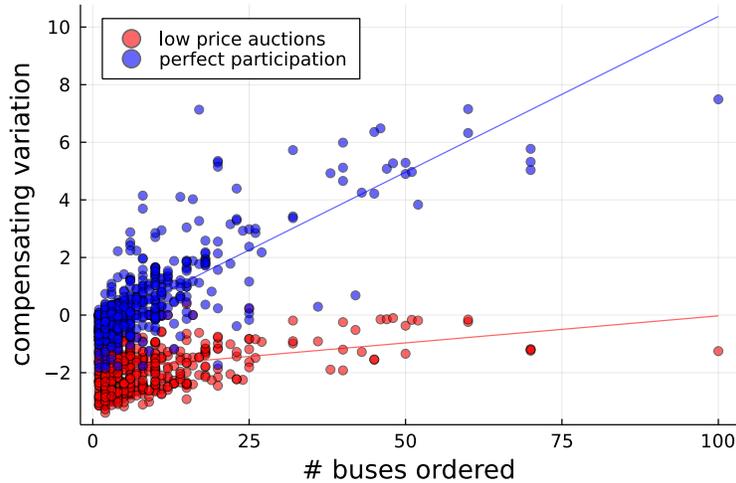


Figure 5: Compensating variation and order size. Solid line depicts a fitted trend.

price auction scenario leads to small decreases in prices, hence the trend of compensation variation is flat. It would take orders of over 100 buses to make the price discount account for the increase in switching costs. In turn, with full participation the price reduction is significant. On average, compensating variation is positive for orders of only 5 or more buses.

9 DISCUSSION

The results of this paper indicate the necessity of accounting for factors other than price in designing procurement markets. The welfare loss associated with eliminating auction favoritism is substantial. A key to improving the situation of the buyers and hence the efficiency of public spending is potential bidder participation. Bid preference programs may successfully balance prices and switching costs if the entry generates enough competitive pressure.

Low auction participation is associated with the lock-in relationship between an operator and a producer. The lock-in weakens participation incentives through two channels. First, the presence of switching costs induces operators to set up a system of preferential weights. Due to entry costs, non-preferred potential bidders do not find it profitable to participate in the auction. Second, incumbent advantage smooths out part of entry costs making them

more likely to participate even without preferential treatment. In addition, participation is low because of the relatively small number of potential bidders across the auctions.

There are a few possible solutions that could mitigate the lock-in effects and encourage more competition without sacrificing fleet unification motive and associated bid preference systems. First, the government may directly subsidize entry. The counterfactual scenario of full participation can be seen as a limiting case of such a subsidy. The idea is to pay a part of potential bidders' entry costs to encourage participation without affecting bid preference, especially among potential bidders with non-favorable weights and non-incumbents that cannot enjoy the benefits of incumbent advantage. However, it may be hard to implement an entry subsidy. An immediate set of questions to ask is who to subsidize—should it be all potential bidders consistently with the principle of equal treatment of the bidders, or should it be potential bidders with non-favorable preference weights? A successful subsidy program would also have to be designed in a way to prevent fictitious entry in which potential bidders would participate only to collect the subsidy, without the intention to bid competitively and win the auction.

Another type of subsidy is to create favorable conditions encouraging more producers to enter the market and increase the pool of potential bidders. However, this type of subsidy faces the risk related to the fact that new producers would initially carry the non-incumbent status in all of the auctions and it may be hard for them to establish their position on the market. As a result, they may be forced to exit the market, making the subsidy wasteful.

Subsidizing costs of participation in an auction or entry to the market are effectively targeting the participation of non-incumbents. Even though it is likely to decrease prices, it may hurt operators' fleet unification efforts and fail to improve market efficiency. Since the low number of potential bidders in auctions is partially related to the fact that producers specialize in specific types of buses, a more effective subsidy may be to support the development of a more diverse pool of products offered by existing producers. This subsidy would induce more competition among the incumbents, giving the promise of lower prices while keeping the pool of incumbent brands constant.

Subsidies are not the only way to improve the efficiency of public procurement with switching costs. The results of the counterfactual analysis indicate that with an increase

in order size, the relative importance of disruption costs decreases. Hence, another idea is to make operators organize auctions for larger orders, perhaps less frequently. To take into account uncertainty regarding future funding as well as the fact that yearly only a fraction of buses is to be replaced, the object of an auction can be the right to deliver buses within the next n orders, for some $n > 1$. Such an auction may be much larger than most of the auctions observed in my data, hence should attract more bidders regardless of incumbency status. At the same time, the switching costs would be of smaller importance compared to the value of the order. As a result, operators may lower the degree of discrimination, as price discounts related to more competitive settings may surpass the costs of the introduction of a new brand to their fleet. Less discrimination would imply even more actual bidders. Since its winner would serve a few consecutive orders, the uncertainty regarding the identity of the winner would decrease compared to the baseline setting. Therefore, larger (but potentially less frequent orders) would allow to mitigate the adverse effects of disruption costs and encourage potential bidders' participation at the same time.

10 CONCLUSION

In this paper, I investigate buyers' trade-off between being offered low prices and avoiding switching costs of introducing a new brand that arises in repeated purchases of durable goods using public procurement procedures. I focus on an empirical example of municipal bus operators in Poland who use a common procurement format of scoring auctions to implement bid preference and favor incumbent bus producers in consecutive orders. I collect detailed data on repeated measurement of auction design linked to operators' fleets that is particularly suitable for identifying the main driving forces of the trade-off: producers' costs of participation in an auction and operator's disruption costs related to the introduction of a new brand and maintaining more diverse fleets.

I develop and estimate a structural model of public procurement with bidder favoritism to quantify the costs. Potential bidders' average entry costs amount to 8.4%-10.3% of project completion costs and significantly discourage participation, especially among bidders with unfavorable treatment assigned by the operator. The results suggest a strong two-sided lock-

in relationship between operators and incumbent producers. To keep their fleets unified, operators are willing to pay over a quarter of the average order value for a win of the most suitable incumbent. Incumbent producers enjoy a so-called incumbent advantage, which decreases their entry costs and makes their participation more likely regardless of bid preference.

The main takeaway from this paper is that the design of public procurement should not only target achieving low prices but also account for other aspects contributing to buyers' welfare. Allowing for bidder favoritism may enable buyers to balance their trade-off in repeated purchases of durable goods. However, a necessary condition is to ensure sufficient participation, which may be difficult given the presence of two-sided lock-in. I discuss several policies that may improve participation and at the same address the lock-in relationship between operators and incumbent producers, based on subsidizing potential bidders' entry, market entry of new potential bidders, and expanding the offer of existing potential bidders, as well as redesigning the timing and scale of organized auctions.

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APPENDIX

A PROCURING CITY BUSES IN POLAND

A.1 PRODUCERS' TECHNOLOGICAL SOLUTIONS – EXAMPLES

As specified in EU directives (2014/24/EU, 2014/25/EU), scoring auctions and associated scoring rules aim to take into account the quality of the goods procured in addition to their price or life cycle costs and ensure in this way the best value for money purchases. Operators choose bus-related criteria to promote specific qualities, fulfilling the intention of lawmakers. However, in many cases, the solution receiving the full score may not be objectively better than non-preferred solutions. For example, windshield division is the most frequently used criterion related to bus technology. Bus windshields can be divided vertically into two halves, making replacement cheaper in case of one-sided damage. However, the dividing column may affect the driver's view. Panoramic (non-divided) windshields offer a better view. However, in case of any damage, replacement gets more costly. Operators assign a full score for both solutions, confirming that neither solution dominates the other. The possibility of applying windshield division is related to how the bus chassis is constructed. Some producers do not offer a divided windshield.

Another frequently used criterion assigns points for either horizontal or vertical alignment of the engine. Using horizontal engines, producers can increase the number of passenger seats. In addition, access to necessary components may be easier. Vertical engines in turn decrease the number of passenger seats, but usually allow for limiting the number of stairs passengers need to climb to take a seat. Again, the alignment of an engine is an essential part of a bus and cannot be easily adjusted. Depending on the operator, both solutions may be given a full score. Figure (6) shows the distinction between described bus characteristics.

A.2 A SIMPLE EXAMPLE OF AUCTION

To understand how the mechanism of assigning bidder preference weights through scoring criteria related to bus technological solutions works, consider a simple example of an auction with three scoring criteria: price, windshield division, and engine alignment. The lowest price

Figure 6: Criteria related to bus technology – examples.



(a) windshield division



(b) horizontal engine alignment



(c) vertical engine alignment

scores 80 points, and more expensive offers receive proportionally less. Buses with their windshields divided receive 5 points, and horizontal engine alignment is worth 15 points. Table (8) presents the details.

Suppose that two bidders submit their offers. S offers cheaper buses, hence gets 80 points in price criterion. Their buses have divided windshields and vertically aligned engines, which results in 5 points assigned for bus-related criteria. M submits a higher price and hence receives fewer points. They offer no windshield division but receive 15 points for a vertically aligned engine. Eventually, S wins with 85 assigned points.

Table 8: Scoring auction design – example.

Auction Design			Bidders			
criterion		max points	S		M	
			value	points	value	points
price	prop. to lowest	80	\$150k	80	\$200k	60
windshield division	yes	5	✓	5	–	–
	no	0	–	–	✓	0
engine alignment	horizontal	15	–	–	✓	15
	vertical	0	✓	0	–	–
total		100	85		75	

In this auction M receives more points than S in bus-related criteria, hence it is more preferred than S by the operator. If M submits a price lower than S, then they surely win. The link between points assigned by the scoring rule and bidder preference weights lies in the question of how much higher price M can submit in comparison to S’s bid and still win. Using the fact that points for price are assigned proportionally to the lowest submitted price, the necessary and sufficient condition for M to win despite bidding a higher price is:

$$\frac{b_S}{b_M} \cdot 80 + 15 \geq 80 + 5 \quad \Rightarrow \quad b_M \leq 1.143b_S$$

The second inequality corresponds exactly to the way we define bidder preference weights θ . The weight for the most preferred bidder M $\theta_M = 1$, and $\theta_S = 1.143$.

B DETAIL ON DATA COLLECTION AND PROCESSING

B.1 DATA COLLECTION

The first step of data collection was to identify auctions. To produce a comprehensive picture of the market, I purchased a list of all auctions that have been concluded between 2011–2019 from a small consulting company operating in the industry. I expanded the list for 2006–2010 and 2020–2022 using Tenders Electronic Daily (TED), a European online service on public procurement¹², tracking new arrivals in operators’ fleets, and browsing industry press¹³.

¹²European Law requires publishing a contract notice at TED if the estimated value of the contract exceeds a certain amount of money. That amounts to approximately 2–3 buses.

¹³Infobus (transinfo.pl/infobus) and Transport Publiczny (transport-publiczny.pl) were particularly useful.

For each auction identified in the first step, I sought official documents to retrieve order requirements, scoring rules, and submitted bids. This data was still available online for more recent auctions. Others came from various sources. The consulting company mentioned has provided me with a significant fraction of the missing documents. I requested the remaining documents directly from the operators. However, some older auctions were not available any longer through this channel. I have managed to retrieve documents for some older auctions by scraping Internet archives¹⁴ and document sharing platforms¹⁵. In total, I collected documents for 85% of all identified auctions between 2006 and August 2022 (97% for auctions in 2011–2022).

The operators' fleets data comes from scraping a webpage <http://phototrans.eu/>, a photo gallery that has evolved into a comprehensive database about vehicles owned by operators all around the world, especially in Poland. For each bus-operator pair, dates of purchase and scrapping (or re-selling), a list of previous owners, and some limited technical details are available. The coverage of the fleet data is very good. In particular, it contains virtually all of the buses listed in auction data.

B.2 DATA PROCESSING

The official procurement documents come in the form of pdf files of various quality. I process them using optical character recognition algorithms to retrieve auction order requirements. However, extracting the definition of criteria within the scoring rule was hard to automate, as each operator formulates them in their way, using different expressions and formats. I process each auction manually to maximize the quality of extracted scoring rules, as they are of central interest to my analysis. For similar reasons, auction results (bidders, bids, final scores) have been processed manually as well.

Auction requirements define a set of potential bidders, that is, a list of producers that offer products of demanded characteristics and therefore can participate in the auction. A proper definition of the set of potential bidders is important to study auction participation. I define the set of potential bidders liberally, taking into consideration demanded length and

¹⁴Wayback Machine - an Internet archive services, web.archive.org

¹⁵docplayer.pl was particularly helpful.

drive of the bus and the year in which the auction is carried. The first two categories define a broad type of bus. The last accounts for producers' entry and exit. I identify potential bidders by analyzing the offer of each producer in a given year based on participation in auctions with a given set of requirements and official product brochures.

I focus my attention on orders for low-entry or low-floor buses, as these constitute a standard in the European urban bus market and are nearly always demanded by municipal operators. Moreover, I restrict the sample to cover orders for buses of lengths exceeding 8 meters. This is motivated by the fact that the smallest vehicles are produced with different technology and mostly by different manufacturers. All of the potential bidders for which I impute the bidder preference weights offer buses only above 8 meters. These account for approximately 90% of all auctions (926).

B.3 DATA IMPUTATION

The official documents provide the total number of points assigned by the scoring rule to participating bidders. This allows me to construct bidder preference weights among participants and use them to solve for optimal bidding. However, to investigate how bidder preference weights affect the bidder's entry, I also need to know the number of points in criteria related to the bus that would be assigned to nonparticipants had they entered an auction. This requires knowledge of particular technological solutions used by producers. I learned them from a range of sources, including official product brochures, results of other auctions in which scoring criteria included a feature of interest and a given producer participated, internet galleries, and YouTube videos¹⁶, and use them to impute the number of points assigned by operators to the non-participating potential bidders.

The imputation proceeds in two steps. First, using knowledge of bus technological solutions by producers, scoring rules announced by operators, and sets of potential bidders across auctions, I develop an algorithm assigning points to the potential bidders. I focus on 65 bus characteristics used as technological criteria (referred to as processed criteria), covering 80% of the total number of points assigned in criteria related to the bus in all auctions and eight

¹⁶For more observable characteristics, for instance counting the number of seats available from the floor level that are easily accessible for disabled passengers.

producers with 95.5% of sales in the auction data. For each auction, potential bidder, and processed scoring criterion, the algorithm chooses a solution offered by the producer that would receive the highest number of points within the criterion. Next, the number of points assigned in this way across processed criteria is summed. If the scoring rule is constructed using only processed criteria, the algorithm returns the sum. If the scoring rule contains criteria outside of the set of 65 processed criteria, the algorithm rescales the obtained sum by the ratio of the total number of points available in all bus criteria to the total number of points available in processed criteria.

There are two key aspects of the imputation process. First, the algorithm needs to accurately assign points for technological solutions. For various reasons, producers may not choose to offer solutions that would bring them the maximum number of points within a given criterion¹⁷. To test the algorithm quality, I compare the true number of points received by auction participants within the processed criteria with the analogous sum imputed by the algorithm. Table (9) presents the results of t-test with the null hypothesis of the same averages. A very large p-value suggests satisfactory quality of the imputation algorithm.

Table 9: Imputation algorithm quality test: comparison of the means.

true	imputed	difference	p-val
16.735 (9.613)	16.672 (9.623)	0.063	0.887

Second, the rescaling procedure implicitly assumes that the processed criteria are chosen at random from the set of all criteria related to bus technology. To test whether rescaling does not introduce bias into the final imputed number of points, I sum the true number of points received by auction participants within the processed criteria, rescale this number and compare it to the true total number of points assigned to the participant. Table (10) presents the results. I cannot reject the null that rescaling is unbiased.

In the last step, I use equation (1) to transform the total number of points assigned to the bidders in technological criteria into the bidder preference weights.

¹⁷There may be a few reasons behind that. For example, the producer may offer a cheaper solution, or offer a technological innovation that was not predicted by the algorithm.

Table 10: Scaling quality test: comparison of the means.

true	scaled	difference	p-val
19.819 (9.615)	19.907 (9.806)	-0.088	0.875

C BIDDING EQUILIBRIUM

C.1 EXISTENCE AND UNIQUENESS

The differential equations in (2) seem difficult to work with, as they involve evaluating the inverse bidding functions at different arguments. Fortunately, it turns out that any low-price sealed bid auction with bidder preference weights can be expressed as an asymmetric low-price sealed bid auction without bidder preference weights, defined on the so-called effective units. To see this, for all j define effective bids as $\tilde{b}_j = \theta_j b_j$ and effective cost as $\tilde{c}_j = \theta_j c_j$. Note that now $\tilde{b}_j \in [\theta_j \underline{b}_j, \theta_j \bar{b}_j,] \equiv [\underline{\tilde{b}}_j, \bar{\tilde{b}}_j]$. Analogously, $\tilde{c}_j \in [\theta_j \underline{c}_j, \theta_j \bar{c}_j,] \equiv [\underline{\tilde{c}}_j, \bar{\tilde{c}}_j]$.

Define also functions $\tilde{\gamma}_j : [\underline{\tilde{b}}_j, \bar{\tilde{b}}_j] \rightarrow [\underline{\tilde{c}}_j, \bar{\tilde{c}}_j]$ for each j in the following way:

$$\tilde{\gamma}_j(\tilde{b}_k) = \theta_j \gamma_j\left(\frac{\tilde{b}_k}{\theta_j}\right)$$

This function is the effective inverse cost function. Note that:

$$\tilde{\gamma}_j(\tilde{b}_j) = \theta_j \theta_j \gamma_j\left(\frac{\tilde{b}_j}{\theta_j}\right) = \theta_j \gamma_j\left(\frac{\theta_j b_j}{\theta_j}\right) = \theta_j \gamma_j(b_j) = \theta_j c_j = \tilde{c}_j$$

Eventually, let $\tilde{F}_j(\tilde{c}_j) = \text{Prob}[\tilde{C}_j \leq \tilde{c}_j] = \text{Prob}[\theta_j C_j \leq \theta_j c_j] = F_j(c_j)$. Then $\tilde{F} : [\underline{\tilde{c}}_j, \bar{\tilde{c}}_j] \rightarrow [0, 1]$ is a well-defined cumulative distribution function of the distribution of effective costs \tilde{c}_j .

We can express the actual bidder j 's profit in terms of the effective units:

$$\pi_j(b_j; c_j, \theta, J^A) = (b_j - c_j) \cdot \prod_{k \neq j} \left[1 - F_k\left(\gamma_k\left(\frac{\theta_j}{\theta_k} b_j\right)\right) \right] = \frac{1}{\theta_j} (\tilde{b}_j - \tilde{c}_j) \cdot \prod_{k \neq j} \left[1 - \tilde{F}_k(\tilde{\gamma}_k(\tilde{b}_j)) \right] \equiv \tilde{\pi}_j(\tilde{b}_j; \tilde{c}_j, J^A)$$

This equation shows that the low-price auction with bidder preference weights can be expressed in terms of an equivalent standard asymmetric auction, defined on the effective units. In particular, there is a 1-1 mapping between the inverse bidding functions and thus equilibrium bidding. This allows us to invoke theoretical results by [Lebrun \(1999, 2006\)](#) who

proved the existence and uniqueness of the equilibrium in IPV asymmetric auctions with possibly different supports of bidders' cost distributions. Notably, the equilibrium is in pure strategies.

[Lebrun \(2006\)](#) also provided a characterization of the equilibrium. It solves the analogous set of ordinary differential equations expressed in effective units:

$$\tilde{\gamma}'_j(b) = \frac{1 - \tilde{F}_j(\tilde{\gamma}_j(b))}{\tilde{f}_j(\tilde{\gamma}_j(b))} \left[\frac{1}{\#J^A - 1} \sum_{k=1}^J \frac{1}{b - \tilde{\gamma}_k(b)} - \frac{1}{b - \tilde{\gamma}_j(b)} \right] \quad (8)$$

Paired with a set of appropriate boundary conditions, equation (8) determines the set of equilibrium bidding functions. Below I provide a characterization of the equilibrium defined on the effective units. The characterization follows [Lebrun \(2006\)](#). It is straightforward to reformulate it into standard units.

C.2 CHARACTERIZATION

C.2.1 RIGHT BOUNDARY CONDITIONS

I start by describing the right boundary conditions. Without loss of generality, order the bidders non-decreasingly with respect to the upper extremity of their effective cost distribution:

$$\tilde{c}_1 \leq \tilde{c}_2 \leq \dots \leq \tilde{c}_{\#J^A}$$

Let also \tilde{b}_j be the highest bid submitted by j in equilibrium. By definition of the inverse cost function, $\tilde{\gamma}_j(\tilde{b}_j) = \tilde{c}_j$.

Suppose that bidder 1 bids \tilde{b}_1 at \tilde{c}_1 . Any other bidder j with cost realization $\tilde{c}_j > \tilde{b}_1$ cannot win and hence may bid any amount larger or equal to the (effective) cost. For consistency reasons, I follow [Krasnokutskaya and Seim \(2011\)](#) and assume that in such a situation player j simply bids their cost \tilde{c}_j .

The degree of competition at the lowest \tilde{c} 's is decisive about shape of right boundary conditions:

- if $\tilde{c}_1 = \tilde{c}_2$ then the upper extremity of the distribution of submitted bids by both bidders 1 and 2 is equal to these costs. Otherwise, any of these two bidders would have a profitable deviation. Moreover, as $\tilde{c}_k \geq \tilde{c}_1$ for any $k > 2$, bidder k also bid \tilde{c}_1 at \tilde{c}_k .

Therefore, the right boundary conditions are:

$$\gamma_j(\tilde{b}) = \tilde{b}, \quad j = 1, \dots, J$$

where $\tilde{b} = \tilde{c}_1$.

- if $\tilde{c}_1 < \tilde{c}_2$, then player 1 is capable of having positive profit at \tilde{c}_1 by setting $\tilde{c}_1 < \tilde{b}_1 < \tilde{c}_2$. Note that at the boundaries of this interval their profit is zero – on the left-hand side the markup is zero, and on the right-hand side the winning probability is zero. In the interior, the profits are positive. Thus, one can show that:

$$\begin{cases} \gamma_1(\tilde{b}) &= \tilde{c}_1 \\ \gamma_j(\tilde{b}) &= \tilde{b}, \quad j = 2, \dots, J \end{cases}$$

where the maximal submitted bid is obtained from¹⁸:

$$\tilde{b} = \arg \max_b (b - \tilde{c}_1) \cdot \prod_{k=2}^J [1 - \tilde{F}_k(b)]$$

It is important to see that if $\tilde{c}_j > \tilde{b}$, then player j has a-priori zero probability of winning. Therefore, favoritism effectively excludes under-preferred bidders with a high-cost realization from an auction.

C.2.2 LEFT BOUNDARY CONDITIONS

To investigate the **left boundary conditions**, relabel bidders such that now:

$$\tilde{c}_1 \leq \tilde{c}_2 \leq \dots \leq \tilde{c}_J$$

As in the case of the right boundary, bidder 1 is the most preferred.

Characterizing the left boundary, [Lebrun \(2006\)](#) distinguishes the lowest possible submitted equilibrium bid \underline{b} but notices that only the first $\kappa(\underline{b})$ bidders actually submit it. For others, it may be too low compared to their minimal cost. Bidders with indices $\kappa(\underline{b})+1, \dots, J$ have their own lower extremities of the distribution of winning bids, \underline{b}_j , $j > \kappa(\underline{b})$. [Hubbard](#)

¹⁸Note that any player j cannot win with a bid larger than \tilde{b} and by assumption bids their cost. Therefore, for $b \geq \tilde{b}$ we get $\tilde{\gamma}_j(b) = b$. So we can skip $\tilde{\gamma}_k$ in the equation.

and Kirkegaard (2019) refer to the phenomenon of unequal lower extremities of the distribution of equilibrium bids submitted by particular players as bid separation.

The lower boundary conditions can be summarized by:

$$\begin{cases} \gamma_j(\underline{b}) &= \underline{c}_j, j \leq \kappa(\underline{b}) \\ \gamma_j(\underline{b}_j) &= \underline{c}_j, j > \kappa(\underline{b}) \end{cases}$$

Values of the lowest bids \underline{b} and \underline{b}_j and $\kappa(\underline{b})$ are not known a-priori. However, Lebrun (2006) shows that the system is completely determined by \underline{b} . Knowing this value, I can solve for the equilibrium bidding.

C.2.3 INTERMEDIATE CONDITIONS

Between the boundaries, the optimal bidding is given by variants of equation (8), depending on bid separation. Bid separation is specific for asymmetric first-price auctions in which lower extremities of cost distributions vary among players and occurs unless $\kappa(b) = J$. With bid separation, there exist a subset of possible equilibrium bid values at their lower range in which only a subset of bidders competes against each other. The bidders realize it, hence equation (8) needs to be altered to account only for bidders who may bid a given \tilde{b} . Each variant of FOC equation is valid in appropriate intervals determined by \tilde{b}_j 's.

Hubbard and Kirkegaard (2019) emphasize the importance of accounting for bid separation. They show in a numerical example that assuming a common lower boundary may lead to completely wrong results. Bid separation is very likely to occur in auctions with bidder preference weights. Even if a researcher is willing to assume the same cost support for all bidders, the presence of weights stretches and shifts the domain of effective cost distributions. Eventually, the problem becomes analogous to an asymmetric auction with varying lower extremities of cost support.

D SHOOTING ALGORITHM IN AUCTIONS WITH BPW

In order to calculate expected profits I need to recover the set of optimal bidding functions $\beta_j(c; \theta, J^A)$ for each auction, configuration of actual bidders, and vector of ex-post bidder preference weights. Equilibrium bidding is a solution to a set of ordinary differential equa-

tions that form a two-point boundary problem, in which the boundary is known only on one side. In addition, the problem is singular at the known boundary. No closed-form solution exists. Numerical tools are needed to solve for equilibrium bidding.

I adapt a standard approach in solving for equilibrium bidding in asymmetric auctions based on shooting algorithms¹⁹ to a setting with arbitrary bidder preference weights. Shooting algorithms are iterative methods of solving boundary value problems that exploit the fact that the solution is fully determined by \underline{b} , the lowest bid submitted in equilibrium. In a standard setting \underline{b} is the lowest bid submitted by all bidders. An additional challenge with bidder preference weights comes from the fact that the lowest bid submitted by a given bidder is not necessarily the lowest bid submitted in equilibrium in general, \underline{b} . Intuitively, discriminatory bidder preference weight may imply that the probability of winning at the lowest realizations of costs is very low, and the under-preferred bidder is actually better off raising their lower bound of the support of bids submitted in equilibrium. This phenomenon is known as bid separation (Hubbard and Kirkegaard, 2019)²⁰. Lebrun (2006) showed that regardless of bid separation, the lowest possible bid submitted in equilibrium \underline{b} still defines uniquely the equilibrium. That makes shooting algorithms particularly suitable for setting with arbitrary bidder preference weights. Guided by Lebrun’s characterization of the equilibrium in a general setting, I adapt the shooting algorithm to account for potential bid bifurcation points²¹.

Hubbard and Kirkegaard (2019) solve for equilibrium bidding with bid separation using methods based on polynomial approximations. However, they consider a special case in which there is a good candidate for a bifurcation point. In a slightly different setting, Bolotnyy and Vasserman (2021) use shooting algorithms accounting for possible bid separation. Both papers consider setting symmetric equilibria with two types of bidders, in which only one bifurcation point can occur. To the best of my knowledge, no other empirical papers mention the possibility of bid bifurcation in their application.

¹⁹The use of shooting algorithms in solving for auction equilibrium has been pioneered by Marshall, Meurer, Richard, and Stromquist (1994) and has been further extended by among others Bajari (2001); Li and Riley (2007); Gayle and Richard (2008). For a review see Hubbard and Paarsch (2014).

²⁰These authors emphasize the importance of accounting for bid separation in solving for optimal bidding, showing in examples that a failure to account for it leads to wrong conclusions.

²¹With N different bidder preference weights we may have up to $N - 2$ bifurcation points.

Some authors express concerns about the instability of shooting algorithms, particularly in the neighborhood of singularity, that is, at the right boundary of project completion costs distribution (Bajari, 2001; Li and Riley, 2007). Fibich and Gavish (2011) provided a theoretical argument for the fact that the instability is not caused by a choice of the algorithm but rather a feature of the problem itself. They show that shooting methods perform worse with an increase in the number of bidders. In my application, the number of potential bidders is relatively low and I find the performance of shooting methods satisfactory.

The separability assumption greatly simplifies the computation. It implies that $\beta_j(c; \theta, J^A, u_t, \Xi_t) = \Xi_t \cdot u_t \cdot \beta_j(c; \theta, J^A, 1, 1)$. Therefore, it is enough to solve for optimal bidding when $u_t = 1$ and $\Xi_t = 1$. I shortly wrote $\beta_j(c; \theta, J^A, 1, 1) = \beta_j(c; \theta, J^A)$.

I use 4th-degree Runge-Kutta methods in solving ODE given each candidate for the boundary.

E BID HOMOGENIZATION

Observed order characteristics x_t may be endogenous in equation (7). If they are correlated with either u_t or b_{jt} , the standard estimation procedure would lead to biased estimates of x_i as well as other parameters in subsequent steps of estimation which rely on homogenized bids. By assumption, u_t is drawn independently from x_t . Problems may arise with b_{jt} , as it is likely correlated with x_t both when $j = 0$ (operator cost estimates) and $j > 0$ (strategic components of bids).

The strategic components of bids b_{jt} for $j \geq 1$ are functions of project completions cost realization c_{jt} , the configuration of actual bidders J_t^A and ex-post bidder preference weights θ_{jt} . The two latter objects are also dependent on ex-ante bidder preference weights $\tilde{\theta}_{jt}$. c_{jt} is uncorrelated with x_t by assumption. However, it is likely the case that more profitable auctions as described by x_t would attract more actual bidders. In turn, the operator may set more discriminatory ex-ante bidder preference weights when the order is small, to avoid fleet fragmentation. With more discriminatory bidder preference weights, the operator may expect higher prices and adjust their cost estimate.

In order to avoid endogeneity bias in estimates of ξ , I control for flexible functions of ex-post bidder preference weights, the number of actual bidders and dummy indicator for

$j = 0$ (denoted by w_{jt}), and order characteristics. Equation (7) becomes:

$$\log b_{jt}^* = x_t' \xi + (w_{jt} \cdot x_t)' \nu + \log u_t + \epsilon_{jt}^b, \quad j = 0, 1, \dots \quad (9)$$

where ξ and ν are vectors of parameters and ϵ_{jt}^b is an error term. I estimate equation (9) using OLS on pooled data containing bids as well as cost estimates b_{0t} . Eventually, I obtain homogenized bids as:

$$\log \hat{b}_{jt} = \log b_{jt}^* - x_t \cdot \hat{\xi} \equiv \log u_t + \log b_{jt}, \quad j = 0, 1, \dots \quad (10)$$

where $\hat{\xi}$ is a vector of estimated parameters ξ . In my application, homogenized bids are sums of unobserved heterogeneity realization and strategic bidding component (or cost estimate draw if $j = 0$).

F UNOBSERVED HETEROGENEITY & PROJECT COMPLETION COSTS

F.1 DETAILS OF ESTIMATION STRATEGY

Homogenized bids combine unobserved heterogeneity and the strategic part of bids. For each auction t , equations (11) for $j = 0, 1, \dots$ can be interpreted as repeated measurements within a measurement error model, in which noisy observed value of $\log \hat{b}_{jt}$ contains a fixed element $\log u_t$ and a random noise $\log b_{jt}$ that are additively separable and independent. This formulation allows me to invoke a statistical result of [Kotlarski \(1966\)](#), who showed that the characteristic function of a sum of two independent random variables is equal to the product of their characteristic functions. Based on this insight, [Krasnokutskaya \(2011\)](#) shows that distributions of unobserved heterogeneity and strategic components of bids are nonparametrically identified from a joint distribution of pairs of repeated measurements, and proposes non-parametric estimators. I adapt her framework to the environment with endogenous entry and bidder preference weights.

[Guerre, Perrigne, and Vuong \(2000\)](#) show that the distribution of project completion costs c is identified from distribution of strategic component of the bids b . To recover the distribution of project completion costs in a setting with entry and bidder preference

weights, it suffices to have an estimate of the distribution of a strategic component of the bids in one configuration of actual bidders and preference weights. The main difficulty in my application lies in the fact that the distribution of the strategic component of bidder's j bid b_{jt} varies with the set of actual bidders J and ex-post bidder preference weights θ_{jt} . In most applications, the latter is drawn from a continuous distribution, which makes it nearly impossible to construct a sample of measurement pairs in which the noise factors follow the same distribution using exclusively $\log b_{jt}$'s. The only exception are single bidder auctions, in which $\log b_{jt}$ depends only on the project completion cost realization. I pair them with the operator's price estimate realization b_{0t} and estimate the distribution of b_{jt} 's on the subsample of single bidder auctions. By assumption, single bidders compete against an auctioneer who draws from cost distribution implied by a distribution of non-preferred bidders G^B . I estimate this distribution in a similar fashion. The underlying project completion cost distribution satisfies:

$$c = b - \frac{1 - G^B(b)}{g^B(b)}, \quad b \sim G^1(b), \quad F^c(c) = G^1(b)$$

The distribution of unobserved heterogeneity cannot be recovered in the same step, as it is likely to affect entry. Instead, I estimate it on a pooled sample of all bids in all auctions with measurement pairs defined in the same way (b_{0t}, b_{jt}) ²².

Mean independence assumption on the distribution of b_{0t} ensures that estimated distributions of u_t and b_{jt} are expressed in the same units regardless of which subsample we use in estimation.

F.2 TECHNICALITIES OF ESTIMATION

The residualized bids combine unobserved heterogeneity and the strategic part of bids:

$$\log bu_{jt} \equiv \log b_{jt}^* - \Xi(x_t) = \log u_t + \log b_{jt} =, \quad j = 0, 1, \dots, J^P \quad (11)$$

²²One may point out that I don't have repeated measurement for auctions with zero bidders. If actual bidders self-select based on values of u_t , missing measurements would cause an identification problem in estimating the distribution of unobserved heterogeneity. However, operators tend to announce new auctions for the same order soon after realizing the previous one attracted no bidders. These auctions are usually exactly the same, operators frequently reuse the same specification. Hence, I assume that in the repeated auction the draw of u_t is the same in both the original and repeated auction. I impute measurements from repeated auctions to the original to control for unobserved heterogeneity in zero-bidder auctions.

In my application, I focus on pairs containing one measurement based on the operator's cost estimate and one measurement related to a submitted bid:

$$\begin{cases} \log bu_{0t} = \log u_t + \log b_{0t} \\ \log bu_{jt} = \log u_t + \log b_{jt} \end{cases}$$

for some j .

Let $\Psi(\cdot, \cdot)$ be the characteristic function of a joint distribution of $(\log bu_{0t}, \log bu_{jt})$ and $\Psi_1(\cdot, \cdot)$ its partial derivative with respect to the first argument. Let $\Phi_u(\cdot)$, $\Phi_0(\cdot)$ and $\Phi_j(\cdot)$ denote characteristic functions of the distributions of unobserved heterogeneity, cost estimates, and bids respectively. Then:

$$\Phi_u(t) = \exp \left\{ \int_0^t \frac{\Psi_1(0, a)}{\Psi(0, a)} da - it\mathbb{E}[\log b_0] \right\} \quad (12)$$

$$\Phi_0(t) = \frac{\Psi(t, 0)}{\Phi_u(t)} \quad (13)$$

$$\Phi_j(t) = \frac{\Psi(0, t)}{\Phi_u(t)} \quad (14)$$

Normalization is needed to uniquely pinpoint the characteristic functions of u , b_0 , and b_j . It is convenient to impose the normalization on the distribution of price estimate b_0 , as it remains the same in each auction, regardless of the number of bidders and realizations of ex-post bidder preference weights. I set $\mathbb{E}[\log b_0] = 0$.

In the next step, densities g^u , g^0 and g^j of $\log u$, $\log b_0$ and $\log b_j$ respectively are retrieved from Fourier inversion:

$$g^u(u) = \frac{1}{2\pi} \int_{-T}^T \exp\{-itu\} \Phi_u(t) dt \quad (15)$$

$$g^0(b) = \frac{1}{2\pi} \int_{-T}^T \exp\{-itb\} \Phi_0(t) dt \quad (16)$$

$$g^j(b) = \frac{1}{2\pi} \int_{-T}^T \exp\{-itb\} \Phi_j(t) dt \quad (17)$$

The densities of u , b_0 and b_j are given by:

$$f^x(u) = \frac{g^u(\log u)}{u} \quad (18)$$

$$f^0(b) = \frac{g^0(\log b)}{b} \quad (19)$$

$$f^j(b) = \frac{g^j(\log b)}{b} \quad (20)$$

$$(21)$$

The estimation follows a nonparametric method pioneered by [Li and Vuong \(1998\)](#). This approach produces uniformly consistent estimates. [Krasnokutskaya \(2011\)](#) provide convergence rates for a version with unobserved heterogeneity. In the first step, I estimate the characteristic function of the joint distribution of $(\log bu_{0t}, \log bu_{jt})$ and its derivative by:

$$\hat{\Psi}(t_1, t_2) = \frac{1}{N} \sum_{i=1}^N \exp \{it_1 \cdot \log bu_{0i} + it_2 \cdot \log bu_{ji}\}$$

$$\hat{\Psi}_1(t_1, t_2) = \frac{1}{N} \sum_{i=1}^N i \log bu_{0i} \cdot \exp \{it_1 \cdot \log bu_{0i} + it_2 \cdot \log bu_{ji}\}$$

In the second step, I recover equations (12 - 18) using $\hat{\Psi}(t_1, t_2)$ and $\hat{\Psi}_1(t_1, t_2)$.

Finally, to get an estimate of the distribution of individual project completion costs I first obtain the cumulative distribution function of bids rationalizing bidding in the single bidder auctions $G_j^B(b) = \int_0^b g_j^B(t) dt$. Then using the insight of [Guerra, Perrigne, and Vuong \(2000\)](#). Using single bidder auctions, for each b I obtain the cost:

$$c = b - \frac{1 - G_j^B(b)}{g_j^B(b)}$$

The distribution of project completion cost satisfies $F^c(c) = F_j^b(b)$.

There is a few technicalities behind this approach. First, in finite samples estimates of fourier transform may oscillate a lot. In order to deal with it I follow [Diggle and Hall \(1993\)](#) and introduce a damp factor:

$$d(t, T) = \mathbb{1}[|t| < T] \cdot \left(1 - \frac{|t|}{T}\right)$$

The estimated densities become:

$$f_u(u) = \frac{1}{2\pi} \int_{-T}^T d(t, T) \exp\{-itu\} \Phi_u(t) dt \quad (22)$$

$$f_0(r) = \frac{1}{2\pi} \int_{-T}^T d(t, T) \exp\{-itr\} \Phi_0(t) dt \quad (23)$$

$$f^j(b) = \frac{1}{2\pi} \int_{-T}^T d(t, T) \exp\{-itb\} \Phi_j(t) dt \quad (24)$$

Another issue is choosing a smoothing parameter T . Large T produces under-smoothing and wiggly, oscillating estimates. T too small over-smooths it. There are two approaches in the literature. One, pioneered by [Li, Perrigne, and Vuong \(2000\)](#) and [Krasnokutskaya \(2011\)](#) relies on choosing T in a way to match moments of estimates distribution with data. Another used by among other [Bonhomme and Robin \(2010\)](#), relies on approximation to the mean integrated square error of the kernel density estimator. I do not have a kernel here, so I use the former. In choosing T I minimize the sum of the square deviation of the estimated mean and variance for u , r , and b separately. I also avoid large values of T that introduce a significant amount of oscillations into estimates.

This procedure also returns bounds of support of the unknown distributions. In theory, the bounds of estimated distributions should be obtained at the stage of Fourier transform – the obtained density should be zero at all points outside of the support. With finite samples, this is never the case. Therefore, I follow the data-driven approach by [Li, Perrigne, and Vuong \(2000\)](#); [Krasnokutskaya \(2011\)](#); [Andreyanov and Caoui \(2022\)](#) extending it to the case with cost estimate and bid. Recall that we observe repeated measurement on $bu_0 = u + r$ and $bu_j = u + b$. In an infinite sample:

$$\min bu_0 = \underline{u} + \underline{r} \quad (25)$$

$$\max bu_0 = \bar{u} + \bar{r} \quad (26)$$

$$\min bu_j = \underline{u} + \underline{b} \quad (27)$$

$$\max bu_j = \bar{u} + \bar{b} \quad (28)$$

$$\max bu_0 - bu_j = \bar{r} - \underline{b} \quad (29)$$

Which makes 5 linear equations in 6 unknowns. The sixth equation comes from my normalization:

$$\int_{\underline{r}}^{\bar{r}} r \cdot f^0(r) dr = 0$$

Analogously to [Krasnokutskaya \(2011\)](#), one can show that this equation has exactly one solution which guarantees that the system has a unique solution. I use sample analog of these equations to obtain support bounds of estimated distributions. [Krasnokutskaya \(2011\)](#) shows that this approach yields consistent estimates when the data comes from a data generating process with unobserved heterogeneity.

G LATENT CLASS MIXING WEIGHTS – PARAMETER ESTIMATES

Table 11: Parameter estimates of σ_3 —mixing weights of latent class distributions

	mixing weights parameters σ_3
incumbent	1.18*** (.24)
brand’s current generation in the fleet	.444 (.29)
won within past year	.936** (.469)
2nd hand delivery within past year	.21 (.409)
won within past 3 yrs	.824** (.354)
2nd hand delivery within past 3 yrs	-.56 (.428)
producer’s share in fleet if wins	-1.4*** (.425)
order size (log)	.497*** (.085)
ordered drive already in the fleet	-.52** (.219)
EU funds	.253 (.182)
compatibility index (non-incumbents)	1.32** (.667)
constant	-1.6*** (.404)

Standard errors in parentheses. p. val: *** ≤ 0.01 , ** ≤ 0.05 , * ≤ 0.1 .

H COUNTERFACTUAL OUTCOMES – NUMERICAL APPROACH

There are two crucial aspects of the economic environment in my application that need to be accounted for in counterfactual analysis. First, even though the decision model is static, the identity of the winner in the current auction affects future outcomes. Second, there is inherent uncertainty regarding the identity of the auction winner associated with project completion costs at the bidding stage, potential bidders’ entry costs, and operators’ shocks related to the choice of ex-ante bidder preference weights. To account for these features, I adopt a specific way of conducting counterfactual analysis. I focus my analysis on paths of auctions. A path is an ordered series of auctions by an operator in which the fleet evolves according to the identities of winners. I maintain the order, timing, and characteristics of auctions from the data to account for factors related to the operator’s demand for new buses that are not accommodated in my model, including the availability of funding, expiration

of the current fleet, and technological progress (e.g., zero-emission drives). For each auction on a path, I obtain the expectation of the value of operator’s utility associated with the order, prices submitted by potential bidders, the number of actual bidders, and winning probabilities. I use the latter to draw the winner’s identity and proceed to the next auction on the path. I simulate paths within an operator²³ and average them to obtain one path of expected outcomes per operator.

²³For nearly 90% of the operators I am able to simulate all counterfactual paths and obtain the probability of their occurrence. For the remaining operators—who organized many auctions during the observation window—I obtain the result by randomly drawing from the set of all possible paths.