

# COSTS OF VARIETY AND PUBLIC PROCUREMENT

THE CASE OF MUNICIPAL BUSES

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## Abstract

I study a public procurement environment in which buyers repeatedly purchase differentiated equipment and where subsequent operation or maintenance costs are lower when acquiring incumbent equipment types, giving rise to conflicting incentives: facilitating competition among potential sellers leads to lower prices while restricting competition among them allows buyers to avoid costs of introducing a new variety. I construct novel data on fleet renewal by municipal bus operators in Poland who use a prevalent scoring auction format to favor incumbent bus producers. I develop and estimate a structural model of auctions with bidder favoritism to quantify the main driving forces of the trade-off. The results indicate that favoritism helps buyers manage the trade-off by reallocating the winning probability toward preferred sellers while keeping the competitive pressure on prices if enough bidders appear at the auction. Increasing order size may increase welfare by stimulating participation and weakening the relative significance of variety.

**Keywords**— auction favoritism, switching costs, public procurement, scoring auctions.

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# 1 INTRODUCTION

Price is often only a fraction of buyers' total costs associated with a purchase. In many markets, buyers experience a range of additional costs affecting them even after the purchase. In particular, firms repetitively procuring horizontally differentiated production inputs, i.e. goods of the same purpose but different make, face costs related to product variety. Introducing a new type of equipment may require integrating it into existing infrastructure, expanding staff utilization, maintenance and repair skills, or increasing storage capacity for spare parts. Vehicles in fleets, machinery in production plants, office equipment, or software are examples of horizontally differentiated inputs that give rise to the costs of variety. The costs of variety cause buyers to face conflicting incentives. Facilitating competition among potential sellers leads to lower prices, while restricting competition among potential sellers allows buyers to avoid the disruption associated with introducing a new type.

Balancing the trade-off between the immediate (price) and long-term costs of purchases is more challenging for buyers required to use public procurement procedures. Although these procedures aim to limit buyers' discretion in choosing the seller, favoring some potential sellers over others is typically possible. Theoretically, with a fixed composition of bidders, favoritism can both lower the procurement costs (McAfee and McMillan, 1989) and enable a buyer to address the trade-off. By favoring incumbent sellers, the buyer increases their chances of winning and provides incentives for non-incumbents to bid more aggressively, mitigating the upward pressure on prices. However, if participation in procurement procedures is costly for sellers, too much discrimination may discourage non-incumbents from submitting bids. If this is the case, favoritism fails to balance the buyer's trade-off and stifles the competition allowing incumbents to enjoy monopolistic rents.

This paper answers the question of how to design a procurement environment that would successfully account for buyer's trade-off without impairing competition. I identify empirically the main mechanisms driving the buyers' trade-off and provide solutions to improve buyers' welfare within the existing institutional framework. To achieve this, I study the behavior of Polish public municipal bus operators who repeatedly procure brand-new buses from bus producers. The operators use a prevalent procurement format of scoring auctions

to implicitly implement bid preference, a scheme of favoring some bidders over others. I construct a novel dataset on auction design linked to the evolution of operators' fleets. I derive and estimate a structural model to quantify three main driving forces of the trade-off: sellers' costs of producing and delivering the goods (project completion costs), potential sellers' costs of participation in the procurement procedure (entry costs), and buyers' disruption costs associated with the introduction of a new type of equipment to the fleet (costs of variety). I use the estimates to perform counterfactual exercises assessing whether implementation of bid preference can balance buyers' trade-off between minimizing price and avoiding costs of variety and suggesting policies to improve buyer's welfare.

Any answer to the main research question has significant implications. It may either point to a shift in the public procurement paradigm by showing that excessively limited discretion brings more harm than gain in terms of welfare or suggest limiting discretion even further as a source of economic inefficiency. It is also relevant from the point of view of public policy. Public procurement of goods and services is a vital part of modern economies, accounting for 10%–20% of GDP in developed countries. Finding ways of promoting competition while considering buyer's costs unrelated to the procurement itself may allow for substantial savings of taxpayer money resulting from a more efficient allocation of public funds.

The case of Polish municipal buses is particularly suitable for studying the trade-off. First, the costs of variety are relevant for bus operators. Heterogenous fleets composed of many brands perform poorer compared to more concentrated ones ([Premik and Yu, 2024](#)), creating incentives for bus operators to unify their fleet. Indeed, data indicates that a typical fleet comprises a small fraction of all bus brands available on the market. Second, although Polish municipal bus operators are not legally allowed to favor or discriminate against potential bidders openly, they implement a system of bid preference implicitly using a prevalent format of scoring auctions. The operators systematically favor incumbent producers, consistent with their motive to avoid the costs of variety. In turn, bus producers who show up in the auctions are mainly the favored ones. Third, the institutional environment creates barriers to entry for new bus producers enabling me to identify the set of potential bidders in every auction. Fourth, the operators renew only a fraction of their fleet at a time, which allows me to follow changes in operators' favoritism towards particular bidders in response to changes in the

fleet and hence distinguish the effects of fleet variety from the effects of unobserved qualities of a match in operator-producer pairs.

The estimates confirm the significance of the three costs driving the buyer’s trade-off. The distribution of project completion costs is characterized by a long left tail elevating gains to the buyer from higher participation of sellers: the more entrants show up in the auction, the higher the probability that one of them draws a low realization of the project completion cost, which would decrease the price regardless of favoritism. However, high entry costs—on average between 8.4% and 10.3% of the project completion cost—significantly discourage participation, particularly among non-favored potential bidders. Lastly, operators are willing to pay up to 28.2% of the average order value for a win of the most suitable incumbent.

Counterfactual experiments show that the costs associated with increasing fleet variety exceed the gains from lower prices due to increased competition for many orders, particularly the smaller ones. Thus, forcibly promoting competition while ignoring the underlying lock-in relationship between buyers and incumbent sellers would be counter-productive. Bid preference programs can balance the trade-off between price and costs of fleet variety by reallocating winning probability towards the favored bidders if enough sellers show up in the bidding. Given these findings, a relatively simple solution to improve the efficiency of public procurement under the presence of the costs of variety is to make buyers purchase more at once. A non-incumbent’s win has a proportionally smaller impact on fleet variety in large orders because their implied fleet share would be higher. As a result, the relative significance of price in the trade-off increases, decreasing incentives to favor the incumbents. Less favoritism in large orders (which are more profitable for the sellers) would likely improve bidder participation, driving down the prices. Therefore, a decrease in both the immediate procurement costs and long-term costs of variety is attainable.

This paper contributes to a few strands of literature concerning buyer’s switching costs, favoritism in auctions, scoring auctions, asymmetric auctions, and low bidder participation in public procurement procedures.

Buyer’s switching costs are extensively studied in the literature ([Klemperer, 1995](#)). More recent applications concern consumer heterogeneity and inertia ([Hortaçsu, Madanizadeh, and Puller, 2017](#); [Miller, Petrin, Town, and Chernew, 2019](#)), transaction costs ([Luco, 2019](#)),

and interactions between adverse selection, regulation and inertia (Polyakova, 2016). Cabral and Greenstein (1990) and Greenstein (1993) showed that switching costs also affect government purchases. A common denominator of these papers is that they study switching costs in posted price environments where agents have full discretion over their choices. Moreover, the switching costs are discrete—refer to fully switching from one alternative to another. To the best of my knowledge, this paper is the first study to comprehensively analyze buyers’ switching costs within the auction environment by identifying trade-offs and suggesting solutions to improve market efficiency. I also offer a continuous interpretation of switching costs through the costs of variety.

Several authors studied applications of bid preference programs that were designed to support domestic or local firms (McAfee and McMillan, 1989; Rosa, 2019), minority and women-owned businesses (Ayres and Cramton, 1996; Mummalaneni, 2022), and small entrepreneurs (Krasnokutskaya and Seim, 2011). Even though bidder discrimination serves predominantly normative purposes in these papers, favoring high-cost potential bidders may decrease procurement costs (McAfee and McMillan, 1989; Corns and Schotter, 1999). My paper shows that auction favoritism can be rationalized by a positive motive: to avoid the costs of variety. I extend the analysis of buyer’s welfare in the auction context by allowing it to depend not only on procurement costs (price) but also on other types of buyer’s costs. I also generalize the standard model of auction favoritism by introducing a random disturbance into assigning bidder preference weights, reflecting possible evaluation shocks.

My paper connects the literature on scoring auctions and favoritism in auctions. The buses are acquired through scoring auctions, a prevalent auction format in the EU (60% of all tenders) and in the US. In scoring auctions, the bids are ranked according to the scoring rule announced by the buyer before bidding listing scoring criteria that assign points in many dimensions of the demanded order. Scoring auctions were originally meant to encourage higher quality of submitted bids (Che, 1993; Asker and Cantillon, 2008, 2010) based on a presumption that potential sellers weigh price and quality in preparing their bids and that ex-ante any bidder is capable of obtaining the full score. However, as the choice of scoring criteria remains at the buyer’s discretion, buyers may deliberately choose criteria that can be satisfied only by a subset of bidders. The bus operators tend to select criteria

related to bus technological solutions that are specific to particular bus producers. In this case, some potential sellers cannot ex-ante obtain the full score for technicalities, so they need to compete more aggressively on price to win. Hence, the scoring auction becomes a discriminatory auction or an auction with bid preference, in which favored bidders enjoy a price advantage over their competitors.

The existing papers studying bid preference consider a setting where there are two groups of bidders—preferred and non-preferred. The bid preference rate is set by the market regulator instead of the buyers and remains the same for all bidders within a group. My application differs substantially, as buyers assign arbitrary favoritism weights to potential bidders. I show that the resulting auction setting is equivalent to a first price auction with asymmetric bidders endowed with cost distributions characterized by arbitrary extremities and use results of [Lebrun \(2006\)](#) to show the existence and uniqueness of equilibrium in the bidding game and provide a characterization of the equilibrium bidding. The characterization implies a possibility of bid bifurcation, which cannot be handled by the standard numerical methods of solving for optimal bidding ([Hubbard and Kirkegaard, 2019](#)). [Bolotnyy and Vasserman \(2023\)](#) account for one possible bid bifurcation point. I adapt the existing numerical methods of solving for equilibrium bidding to account for arbitrary patterns of bid bifurcation that may occur due to arbitrary bidder preference weights.

Lastly, it is a well-documented fact that public procurement procedures often attract few bidders. Over 24% of all auctions in the EU observe a single bidder ([Titl, 2021](#)), and 45% of the value of federal contracts in the US has been assigned in a single bidder setting ([Kang and Miller, 2022](#)). In my empirical application, this number exceeds 50%. Low participation is attributed to incumbent’s cost advantage ([Iossa, Rey, and Waterson, 2022](#)), insufficient publicity and information about the contracts ([Coviello and Mariniello, 2014](#)), corruption and political connections ([Baranek and Titl, 2024](#)), natural monopoly. This paper adds to this list by showing that low participation can also be attributed to buyers’ attempts to avoid costs of variety associated with introducing a new brand into the fleet.

The remainder of this paper is structured as follows. Section 2 describes the economic and institutional environment in which municipal bus operators in Poland purchase their buses. In section 3, I introduce data and discuss the main patterns. Section 4 introduces

the model. Section 5 outlines the estimation strategy and presents the results. In section 6, I use the estimated model to perform counterfactual exercises. Section 7 concludes.

## 2 ECONOMIC AND INSTITUTIONAL ENVIRONMENT

This section motivates the main trade-off studied in this paper. I also describe the environment in which Polish municipal bus operators procure new buses using the scoring auction format, which enables them to favor some bidders over others.

### 2.1 COSTS OF VARIETY

Urban bus operators produce transit services using labor and capital inputs. The former includes mainly drivers and mechanics, and the latter comprises almost entirely buses. Even though all urban buses serve the same purpose of transporting passengers, they are highly differentiated goods. Numerous bus producers on the market have developed distinct technological ways of producing buses. As a result, the bus's physical structure and components vary across brands, which induces differences in how vehicles need to be operated, maintained, and repaired.

This horizontal differentiation has important implications for the cost structure of a bus company. First, introducing a qualitatively different technology may require costly adjustments to accommodate it within the existing infrastructure. This may include, among others, training drivers and maintenance staff, establishing fueling sites for a new type of fuel, or setting up channels of spare part delivery. Second, holding a heterogeneous fleet with many different bus types at the depot may generate costs associated with maintaining various sets of skills among the employees and increased demand for other spare parts. These costs of variety are likely to persist as long as the degree of fleet diversity remains unchanged.

The fleet variety affects the productivity of bus operators' production inputs. Using data on the same Polish municipal bus operators, Premik and Yu (2024) show that buses of the same type drive systematically more miles per year in more unified fleets than in fleets with a higher degree of heterogeneity. Moreover, the labor input is also more productive among

the former. Similar results are obtained in other markets facing horizontal capital input differentiation, including the airline industry ([Merkert and Hensher, 2011](#); [Merkert, 2023](#)).

The costs of variety create potentially strong incentives for fleet unification. A bus operator may prefer to pay a higher price while purchasing buses to ensure they are of a type already existing in their fleet. This motivation is stronger when the purchase concerns few buses relative to the fleet size. All of the above reflects the essence of the buyer's trade-off between the costs of procurement and the costs of variety, the main focus of this study.

## 2.2 MARKET ENVIRONMENT

Poland is a relatively big market for municipal buses. At each point in time, there are around 10 producers who supply operators with new buses. The exact numbers vary with producers' entry and exit. The producers are usually well-established, regular participants in the market. High producer's entry costs, including obligatory vehicle certification and setting up a nationwide system of affiliated workshops, significantly limit fringe bidding. Bus producers sell their products to approximately 200 municipal operators holding a stock of over 12,000 vehicles. The demand for transportation services in Polish cities remains relatively stable, hence the total number of vehicles within a firm does not vary significantly. Operators have buses of different ages and renew their fleet gradually through small orders. Bus operators own their fleets. In most cases, they are also the ones to perform maintenance and repairs. It is relatively rare for them to lease the vehicles. The vast majority of bus operators are either owned or financed by local authorities. That imposes a legal requirement to use public procurement for larger purchases to ensure transparency of public fund spending. The procurement format is the scoring auction, mandated by national and European procurement laws.

## 2.3 PROCUREMENT OF NEW BUSES

An auction is announced when the operator publishes an auction specification, a set of documents specifying order requirements. It also includes a definition scoring rule, which is an algorithm that serves to rank submitted bids. The scoring rule contains a list of criteria



related to the order that are preferred but not required by the operator. Each criterion assigns several points to a bid that satisfies it. For each bid, points from different criteria are added up to a maximum of 100 points that an offer may receive. The winner is a bidder whose offer gets the largest number of points.

After the announcement, potential bidders are given one to three months to prepare their bids. During this time, they are allowed to ask the operator questions regarding the specifics of the order. The dialog between potential bidders and the operator is published with the auction specification. Despite questions being asked anonymously, they often refer to particular technological solutions specific to some producers. Interviews with people from the industry suggest it is easy to infer the identities of producers asking the questions. Next, bidders submit their bids in secret. The bids are opened and ranked by an operator using the scoring rule. Eventually, the winner is announced. The operator publishes auction results including the total number of points assigned through the scoring rule for each bidder.

## 2.4 SCORING AUCTIONS AND BIDDER PREFERENCE WEIGHTS

A profile of bidder preference weights  $\theta = \{\theta_j\}_j$  is a vector of positive numbers, one for each potential bidder. Operators assign them to favor or discriminate against bidders. Bidder preference weights affect the ranking of submitted bids. The winner is a bidder who submits the lowest value of a product of bid and the weight:

$$j \text{ wins} \Leftrightarrow \arg \min_k \theta_k b_k$$

where  $b_k$  is a price submitted by bidder  $k$ . Therefore, bidder preference weights give an ex-ante price advantage to preferred bidders. It is no longer necessary for the winner to be a bidder who submits the lowest price. Bidder preference weights require normalization. For convenience, I normalize the lowest  $\theta_j$  – the weight assigned to the most preferred bidder – to 1.

Polish bus operators are not legally allowed to discriminate against potential bidders openly. Instead, the commonly used scoring auction format creates an opportunity to implement bidder preference weights implicitly. A key element of the environment is the scoring

criteria used for bid evaluation. Scoring criteria can be divided into two groups: criteria related to the bus and criteria related to the offer (but not directly to the offered bus itself). The latter includes factors such as price<sup>1</sup>, post-sale services or shorter deadlines<sup>2</sup>. Hence, any bidder can get the maximum number of points. The former contains criteria assigning points for specific technological solutions offered by a producer. Producer’s technology rarely can be updated in the short run to get more points in a given auction. Therefore, points assigned in this category are fixed and act as if they have been assigned for the bidder’s identity.

Bidder preference weights are constructed from the number of points assigned to particular potential bidders in criteria related to the bus and the total number of points in offer criteria. Let  $\bar{y}$  be the maximum number of points possible to be assigned throughout criteria related to the offer and  $\rho_j$  be the number of points assigned to the bidders within bus criteria. Without loss of generality, assume  $\rho_1 \geq \rho_2 \geq \dots \geq \rho_J$  and normalize the bidder preference weight for the most preferred bidder to one:  $\theta_1 = 1$ . Then:

$$\theta_j = \frac{\bar{y}}{\bar{y} - \rho_1 + \rho_j}, \quad j \geq 2 \tag{1}$$

This formulation implies that bidder  $j$  wins if and only if  $\theta_j \cdot b_j \leq \theta_k \cdot b_k$  for all  $k \neq j$ . Therefore  $\theta_j$ ’s are bidder preference weights.

Appendix ?? describes a few frequently used scoring criteria related to bus technology and provides a simple example of an auction, including the construction of the scoring rule, points assignment to potential bidders, and the relation between the number of points assigned in technological criteria and bidder preference weights.

Bidder preference weights  $\theta$  as formulated above have interpretation in terms of the advantage of the most preferred bidder over the competitors. For example, if bidder  $j$  is assigned  $\theta_j$ , the most preferred bidder may submit a bid up to  $\theta_j$  times larger than  $j$ ’s bid

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<sup>1</sup>Price is the only criterion that appears in all auctions and is always of greatest importance with a maximum of 68 points on average to be assigned. Auctions with price as the only criterion (equivalently 100 points for price) boil down to standard procurement auctions.

<sup>2</sup>If there are more offer-specific criteria besides price, the price can be easily adjusted to form a single index reflecting also the value of other offer-specific criteria. Moreover, the bus producers often offer the most preferred levels within the offer-specific criteria, even in the single-bidder auctions. Hence, it does not seem that the offer-specific part has a strategic importance in the empirical application.

and still win with  $j$ . We can also reformulate the weights to have an interpretation in terms of  $j$ 's position. Let:

$$\omega_j = \left(1 - \frac{1}{\theta_j}\right) \cdot 100\%$$

$\omega_j$  describes the minimal relative discount that bidder  $j$  needs to make compared to the most preferred bidder to win with them. This formulation seems more intuitive in interpretation. In the remainder of the paper, I use the term bidder preference weights to refer to  $\omega$ 's, unless otherwise explained.

### 3 DATA

In this section, I discuss the data sources and present raw summary measures illustrating the mechanisms governing Polish municipal bus operators' fleet renewal via auctions with bidder preference weights.

#### 3.1 SOURCES

There are two main sources of data used in this paper: auction data and fleet data.

**AUCTION DATA.** The auction data consists of the official procurement documents of 925 auctions by 189 public bus operators organized between 2006 and 2022. I have collected the documents from various sources, including a small consulting company operating in the industry, through direct requests to the operators, or scraping online resources such as Internet archive services (Wayback Machine: [web.archive.org](http://web.archive.org)) and document sharing platforms ([docplayer.pl](http://docplayer.pl) was particularly helpful). For each auction, I process the official documents to retrieve order characteristics, scoring rules, as well as identities, bids, and total score evaluation for participating producers.

Auction characteristics define a set of potential bidders: a list of producers that offer products of demanded characteristics and, therefore, can participate in the auction. I define the set of potential bidders liberally, considering the demanded length and drive of the bus and the year in which the auction is carried. The first two categories define a broad

type of bus with respect to its purpose. The last accounts for producers' entry, exit, and the evolution of their products. I identify potential bidders by analyzing the offer of each producer in a given year based on participation in auctions with a given set of requirements and official product brochures.

The official documents provide the total number of points the scoring rule assigns to participating bidders. This allows me to construct bidder preference weights among participants and use them to solve for optimal bidding. However, to investigate how bidder preference weights affect the bidder's entry, I also need to know the number of points in criteria related to the bus that would be assigned to nonparticipants had they entered an auction. This requires knowledge of particular technological solutions used by producers. I learned them from a range of sources, including official product brochures, results of other auctions in which scoring criteria included a feature of interest and a given producer participated, Internet galleries, and YouTube videos. I use the knowledge about bus characteristics to impute the number of points operators assign to the non-participating potential bidders. Online Appendix B provides a more detailed description of data collection and processing and discusses the quality of bidder preference weights imputation for non-participating potential bidders.

**FLEET DATA.** The operators' fleet data comes from scraping a webpage <http://phototrans.eu/>. This photo gallery has evolved into a comprehensive database of vehicles owned by operators worldwide, with excellent coverage of Polish fleets. In particular, it contains every bus implied by the auction data. For each bus-operator pair, dates of purchase and scrapping (or re-selling), a list of previous owners, and some limited technical details are available.

## 3.2 DESCRIPTIVE ANALYSIS

Table (1) describes the main features of operators' fleets and the auctions they design. Operators tend to unify their fleet, that is limit the number of distinct vehicle brands in possession. In total, operators drive buses of 60 different brands. However, an average individual fleet contains vehicles of approximately four different brands, of which only two are still offered on the market. The distribution of brands within a fleet is not homogeneous.

To investigate it, I use the Herfindahl-Hirschman Index (HHI) defined as a sum of squared shares of brands within a fleet. An average fleet HHI amounts to 0.44, suggesting the presence of approximately two leading brands. To see this, compare this result with HHI for a fleet with two brands of equal share, which equals 0.5. These are likely to be the two brands that are still offered and can expand their fleet share in consecutive purchases. In turn, market-level HHI suggests the presence of approximately five significant players.

Fleet unification is not perfect, meaning that an operator with a single-brand fleet is rarely observed. A single-brand fleet is nearly impossible to achieve due to a few factors. First, producers' entry and exit from the market shape brand availability and lead to an increase in fleets' diversity. Second, operators may buy second-hand buses from abroad that are not sold as brand new on the market<sup>3</sup>. Third, producers tend to manufacture only a subset of possible bus types. Producer specialization is an important source of fleet diversity, especially among operators with more heterogeneous needs regarding bus type.

These considerations reference Harold Zurcher, a superintendent of maintenance at the Madison Metropolitan Bus Company, in the seminal paper by Rust (1987). His fleet contained 162 buses of three brands. One brand (Chance) consisted of only four vehicles of special purpose, likely illustrating the impact of producer specialization. Another brand (Grumman), with 15 buses, has been introduced in the last purchase within the observation window. The remaining brand (GMC) was the main component of Zurcher's fleet and had been the only brand delivered across the 1970s.

Operators tend to make frequent but small purchases. They renew 13.5% of their fleet once in 1.7 years. This reflects purchase patterns from the past and the current availability of funding. Neither purchasing format (auction) nor small but frequent purchases seem helpful in achieving fleet unification. Despite this, operators manage to keep the number of distinct brands low. Bidder preference weights are key to solving this apparent puzzle.

Operators use bidder preference weights to signal who they want to win. Bidder preference weights in table 1 are expressed in  $\omega$  formulation: they describe how much discount a less preferred bidder has to offer compared to the most preferred bidder to win with them.

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<sup>3</sup>This used to be particularly prevalent among Polish municipal bus operators when funding sources were scarce, up until the second half of the 2000s. In the case of second-hand purchases, nearly immediate availability and low price of these buses were usually the main factors leading to purchases.

Table 1: Operators and their auctions – summary statistics.

	count/mean	st. deviation
Operators		
# operators	189	–
fleet size	75.1	147.5
# of brands – total	60	–
# of brands – in a fleet	3.9	1.8
# of brands – in a fleet, active bidders	2.2	1.3
market fleet HHI	0.21	0.05
operator’s fleet HHI	0.44	0.26
Auctions		
# auctions	925	–
order frequency – years	1.7	2.1
order size – % of the fleet	13.5	17.0
# potential bidders	3.5	1.4
# actual bidders	1.4	0.9
Bidder Preference Weights $\omega$		
1st most preferred bidder	0	0
2nd most preferred bidder	5.5	8.0
3rd most preferred bidder	10.3	10.5
incumbents	3.7	7.2
incumbent who won last auction	2.1	5.6
non-incumbents	9.2	11.4
participants	2.1	5.1
non-participants	10.3	11.4

The weight for the most preferred bidder is, by definition, normalized to 0. An average bidder preference weight assigned to the second-most preferred bidder is 5.5. That means they need to submit a bid 5.5% lower than a bid by the most preferred bidder to win with them. Interpretation of these numbers is conditional on the fact that the most preferred bidder participates in the bidding.

Operators favor producers that are already in their fleet. An average incumbent has to submit a bid lower by 3.7% than the most preferred bidder to win. Non-incumbents need, on average, to offer a 9.2% price discount. Data reveal that an average incumbent is not the most preferred bidder. This observation may be a result of various factors. Some operators may dislike the products of particular incumbents due to past bad experiences. Others may have discovered other brands that better fit their needs. It can also be optimal to maintain a

roughly equal share of a few brands to insure against faulty products and be able to attract more bidders in auctions. All of these are consistent with the more preferential treatment assigned to the most recent winner (necessary discount of only 2.1%), which is likely to describe the current operator’s preferences most closely. Bus operators tend to continue buying from the same sources. Similarly, Harold Zurcher’s company bought exclusively from GMC throughout the 1970s.

Bidders’ auction participation heavily depends on bidder preference weights. A typical participant is either the most preferred or nearly the most preferred bidder. The average bid discount required to win among the entrants is 2.1%. In turn, non-participants would have needed to offer a discount of more than 10%.

Bidder preference weights affect the producer’s chance of winning in two ways. They force under-preferred bidders to bid lower if they decide to bid. However, if participation in an auction is costly, the markup margin may not be large enough to accommodate an inferior bidding position resulting from bidder preference weights. This is why a typical auction could attract more than three bidders, yet two of them do not decide to submit a bid.

## 4 MODEL

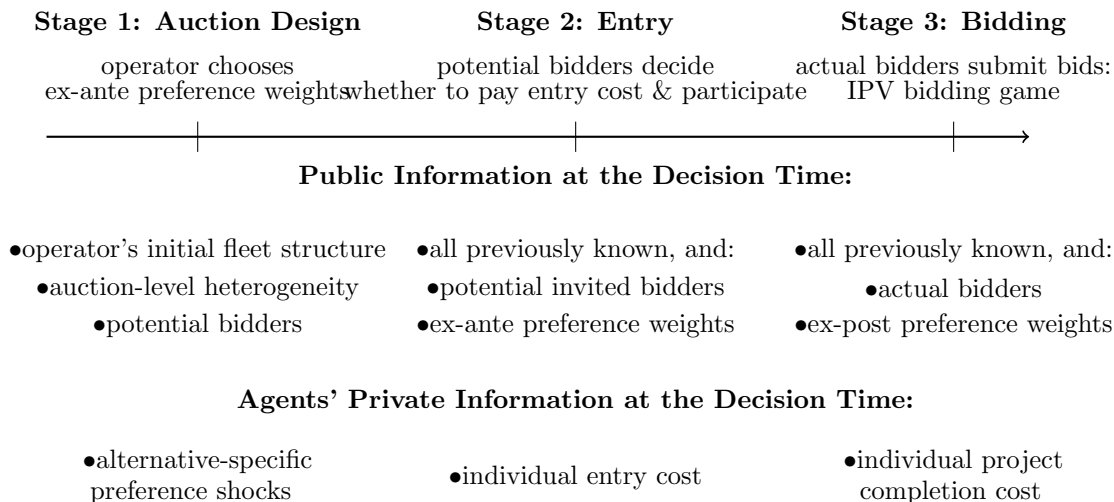
In this section, I develop a model of repeated interactions between a single buyer (bus operator) purchasing differentiated durable goods (buses) from sellers (bus producers) through auctions with favoritism driven by the identities of past winners (fleet structure).

Every interaction encompasses four stages. In stage 0, the nature draws order characteristics that determine the set of potential bidders. In stage 1, considering the current fleet structure, the buyer chooses a potentially discriminatory preference weight for each potential bidder. In stage 2, the potential bidders decide whether to pay an entry cost enabling them to participate in the auction. In stage 3, actual bidders—potential bidders who chose to participate in bidding and paid their entry costs—submit bids.

Figure (1) summarizes the timeline of decision stages of the game and information available for the agents at every decision node. In the remainder of this section, I describe the

model and discuss the modeling assumptions through the lenses of my empirical application. However, it is straightforward to adapt the model to fit any other market in which the fact that the identity of the procurement winner matters for the buyer’s welfare induces favoritism in auction design.

Figure 1: Decision stages of the game.



## 4.1 MARKET DEFINITION

The economy is populated by bus operators collected in the set  $I$  (with a typical representative  $i$ ) and bus producers collected in the set  $J$  (with a typical representative  $j$ ). Being located in a separate city, every bus operator defines an independent market. The model focuses on a single procurement event  $t$  interacting operator  $i$  with bus producers in  $J$ . Assume that all bus operators and bus producers are risk-neutral.

## 4.2 STAGE 0: ORDER CHARACTERISTICS

The game begins with the move of nature choosing characteristics of the order, included in vector  $x_{it} \in X$ , where  $X$  is a set of all possible relevant order characteristics. The order characteristics are higher-level procurement features, such as the number of the buses ordered and their purpose (defined by the type of drive, e.g., conventional or electric, or their size).



The auction characteristics are observed by every agent in the game and the econometrician. I refer to  $x_{it}$  also as to the observed auction-level heterogeneity.

Since  $x_{it}$  is chosen by nature, it is essentially exogenous in the model. In reality, bus operators may affect the specifics of the order. However, the type of buses ordered typically reflects the size of the city and hence is not strategic. Moreover, the order characteristics are also driven by the availability of external funding, which is the primary driving force for public bus operators in Poland to purchase new buses. The funding determines the size of the order, and the fund provider may request purchasing buses of specific characteristics, for example, powered by an electric powertrain. These factors justify the exogeneity assumption.

The observed auction-level heterogeneity determines the expected cost of delivering the order, denoted by  $\Xi(x_{it})$ , affecting the expected bidder payoffs. Since bus producers tend to specialize in producing only a subset of all available bus types,  $x_{it}$  also defines the set of potential bidders in the procurement: bus producers who can deliver the requested order. Denote the set of potential bidders in the current auction by  $J_{it}^P \equiv J(x_{it})$ .

### 4.3 STAGE 1: AUCTION DESIGN

With the definition of auction characteristics, the game proceeds to the stage in which the auctioneer chooses the auction design.

**INFORMATION STRUCTURE.** The buyer observes their initial fleet structure, which is summarized by the set of variables  $\varphi_{it}^0 = \{\varphi_{ijt}^0\}_{j \in J}$  describing contributions to the fleet by every existing bus producer. They include, among others, the number of buses of a specific brand, the interaction of the brand with other bus features, and the history of past deliveries. Importantly,  $\varphi_{it}^0$  does not include vehicles that will be replaced by the procured buses and scrapped upon their arrival. Lastly, the operator also observes a set of alternative-specific preference shocks  $\epsilon$ , which I describe in detail after introducing buyer's choice set.

**DECISION VARIABLES.** The operator designs the auction by choosing a profile of ex-ante bidder preference weights:

$$\tilde{\theta}_{it} \equiv \{\tilde{\theta}_{ijt} \in \tilde{\Theta} : j \in J_{it}^P\}$$

assigning each potential bidder with an ex-ante bidder preference weight from the set of all possible ex-ante bidder preference weights  $\tilde{\Theta}$ . I assume that  $\tilde{\Theta} = \{0, 1, 2\}$  denoting most preferred bidders, less preferred bidders, and non-preferred bidders<sup>4</sup>.

The ex-ante bidder preference weights describe the intended level of favoritism in the auction. They are possibly noisy signals of the ex-post bidder preference weights  $\theta_{ijt}$  used to rank the submitted bids at the bidding stage. The separation of ex-ante and ex-post bidder preference weights generalizes the standard approach in the literature on auctions with bidder preference weights and endogenous entry (Marion, 2007; Krasnokutskaya and Seim, 2011; Mummalaneni, 2022), in which  $\tilde{\theta}_{ijt}$  coincides with  $\theta_{ijt}$ . It is motivated by the literature concerning settings that allow for randomness in the bid evaluation process (Takahashi, 2018; Krasnokutskaya, Song, and Tang, 2020).

The ex-ante bidder preference weights affect the expected price submitted by the bidders and their winning probabilities. Hence, they are tools allowing bus operators to probabilistically balance the trade-off between the procurement costs (measured by the price) and life-cycle costs of maintaining the future fleet structure.

**OBJECTIVE FUNCTION.** Consider an event in which bidder  $j \in J_{it}^P$  wins by submitting a bid (price)  $b_{ijt}$ . The arrival of new buses produced by  $j$  causes the update in  $j$ 's contribution to the fleet  $\varphi_{ijt}^1 \equiv \varphi_{ijt}^1(\varphi_{ijt}^0, x_{it})$ , which is a deterministic function of the  $j$ 's initial fleet contribution and order characteristics. The updated fleet structure is expressed as:

$$\varphi_{it|j}^1(\varphi_{it}^0, x_{it}) \equiv \{\varphi_{i1t}^0, \dots, \varphi_{ijt}^1, \dots, \varphi_{iJt}^0\}$$

To simplify the notation, define the relevant state variables as functions of the fleet structure and the order characteristics  $z_{ijt} = \zeta(\varphi_{ijt}^0, x_{ijt})$ . If potential bidder  $j$  wins auction  $t$ , the operator  $i$  receives a stream of utility:

$$-\alpha b_{ijt} + \gamma_{ij} z_{ijt}$$

---

<sup>4</sup>Given the relative nature of the weights, I assume that there is at least one most preferred bidder in each auction. Hence, the operator chooses a vector of ex-ante bidder preference weights from  $|J_{it}^P| \cdot |J_{it}^P| \cdot 3^{|J_{it}^P|-1}$  possible choices. For example, a typical auction with three potential bidders generates  $3 \cdot 3^2 = 27$  possible choices.

where  $b_{ijt}$  is the price implied by the winning bid. The payoff to the auctioneer if there is no winner is normalized to zero for identification reasons.

The outcome of a procurement is determined stochastically through the auction mechanism, which depends on the vector of ex-ante bidder preference weights  $\tilde{\theta}_{it}$ . Operator's expected stream of utility given a choice of  $\tilde{\theta}_{it} \in \tilde{\Theta}$  is:

$$\sum_{j \in J_{it}^P} \varrho_j(\tilde{\theta}_{it}) \cdot (-\alpha \mathbb{E}[b_{jt} | \tilde{\theta}_{it}] + \gamma_{ij} z_{ijt}) \equiv -\alpha \hat{b}_{it}(\tilde{\theta}_{it}) + \hat{z}_{it}(\tilde{\theta}_{it}) \quad (2)$$

where  $\varrho_j(\tilde{\theta}_{it})$  is the probability that potential bidder  $j$  wins given  $\tilde{\theta}_{it}$ , and  $\mathbb{E}[b_{ijt} | \tilde{\theta}_{it}]$  is the expected price submitted by bidder  $j$  given  $\tilde{\theta}_{it}$  and the fact that  $j$  wins, and  $\hat{b}_{it}(\tilde{\theta}_{it}) = \sum_{j \in J_{it}^P} \varrho_j(\tilde{\theta}_{it}) \cdot \mathbb{E}[b_{jt} | \tilde{\theta}_{it}]$  and  $\hat{z}_{it}(\tilde{\theta}_{it}) = \sum_{j \in J_{it}^P} \varrho_j(\tilde{\theta}_{it}) \cdot \gamma_{ij} z_{ijt}$

Eventually, I assume that each choice of  $\tilde{\theta}_{it}$  is associated with a random shock  $\varepsilon(\tilde{\theta}_{it})$  drawn independently and identically across alternatives and auctions from the extreme value type 1 distribution. The operator's decision problem is:

$$\max_{\tau} \left\{ -\alpha \hat{b}_{it}(\tau) + \hat{z}_{it}(\tau) + \varepsilon(\tau) : \tau \in \prod_{j \in J^P} \tilde{\Theta} \right\}$$

The probability that operator  $i$  chooses a profile of ex-ante bidder preference weights  $\tilde{\theta}_{it} \in \tilde{\Theta}$  in auction  $t$  can be written as:

$$P[\tilde{\theta}_t = \tilde{\theta} | \zeta] = \frac{\exp\{-\alpha \hat{b}_{it}(\tilde{\theta}_{it}) + \hat{z}_{it}(\tilde{\theta}_{it})\}}{\sum_{\tilde{\theta}'_{it} \in \tilde{\Theta}} \exp\{-\alpha \hat{b}_{it}(\tilde{\theta}'_{it}) + \hat{z}_{it}(\tilde{\theta}'_{it})\}} \quad (3)$$

**EQUILIBRIUM.** The operator problem is a single-agent discrete choice utility maximization problem with alternative-specific shock distributed on the real line. Hence, the optimum choice exists and is well-defined.

#### 4.4 STAGE 2: ENTRY

In the second stage,  $J_{it}^P$  potential bidders decide whether to pay an entry cost and participate in the auction.

INFORMATION STRUCTURE. The profile of ex-ante bidder preference weights  $\tilde{\theta}_{it}$  chosen by the buyer in the auction design stage is common knowledge among agents in the market. However, the econometrician does not observe it, as the data typically contains only the realized ex-post bidder preference weights.

In addition, each potential bidder draws a private realization of an entry cost  $e_{ijt}$ :

$$e_{ijt} \sim F_j^e(\cdot) : \mathcal{D} \rightarrow [0, 1]$$

where  $F_j^e$  is a continuous cumulative distribution function and  $\mathcal{D}$  is a compact subset of  $\mathbb{R}$ . Each invited potential bidder knows their entry cost realization and the distributions of their competitors' entry costs. The entry costs summarize the effort required to submit an offer, including designing product customization, preparing necessary documentation, securing deals with subcontractors, and the opportunity cost of not participating in the auction. Idiosyncratic variation in entry costs is also attributed to differences in relations with the financial sector (proofs of financial capacity to realize the order).

Potential bidders also observe a realization of  $u_{it}$ , a uni-dimensional index of unobserved auction heterogeneity. The shock is realized at the beginning of the entry stage as an *iid* draw from a continuous distribution  $F^u$  with bounded support and non-vanishing density:

$$u_{it} \sim F^u(\cdot) : [\underline{u}, \bar{u}] \rightarrow [0, 1]$$

I assume that all the actual bidders observe the shock perfectly and that the econometrician does not observe the shock.  $u_{it}$  contains shocks to project completion costs common to all bidders and explains the within-auction correlation of submitted bids.

DECISION VARIABLES. The entry decision is binary. Let  $d_{ijt} = 1$  if bidder  $j$  decides to participate, and zero otherwise.

OBJECTIVE FUNCTION. Auction participation necessitates payment of the entry cost. In return, entrant  $j$  receives an opportunity to submit a bid and win the auction, which yields

a payment of the proposed bid. Implicitly, it also includes a chance to update the fleet for future orders.

The invited potential bidders decide whether or not to pay the entry cost to maximize the expected profit at the entry stage:

$$d_{ijt} = \arg \max_{\{0,1\}} \{0, \pi_j(\tilde{\theta}_{it}, \varphi_{it}^0, x_{it}, u_{it}) - e_{ijt}\}$$

where  $\pi(\tilde{\theta}_{it}, \varphi_{it}^0, x_{it}, u_{it})$  is the expected payoff from participating in the auction given the profile of ex-ante bidder preference weights  $\tilde{\theta}_{it}$ , the initial fleet contribution  $\varphi_{ijt}^0$ , the observed auction-level heterogeneity  $x_{it}$ , and the unobserved auction-level heterogeneity  $u_{it}$ <sup>5</sup>. The expected payoff from participation depends on  $j$  to reflect potential asymmetries among bidders at the bidding stage.

**EQUILIBRIUM.** An equilibrium of the entry stage is a set of optimal entry rules  $d_{ijt}(e_{ijt}; \tilde{\theta}_{it}, \varphi_{it}^0, x_{it}, u_{it})$ , one for each of the potential bidders  $j \in J_{it}^P$ , mapping the realizations of the individual entry costs and the remaining state variables into a binary decision of whether or not to participate in the bidding stage.

The equilibrium entry strategy takes the form of a cut-off rule in which an invited potential bidder enters if:

$$d_{ijt}(e_{ijt}; \tilde{\theta}_{it}, \varphi_{it}^0, x_{it}, u_{it}) = 1 \Leftrightarrow e_{ijt} \leq \pi_j(\tilde{\theta}_{it}, \varphi_{it}^0, x_{it}, u_{it}) \quad (4)$$

The cut-off rule implies that the probability that  $j$  enters the auction is  $p_j(\tilde{\theta}_{it}, \varphi_{it}^0, x_{it}, u_{it}) \equiv F_j^e(\pi_j(\tilde{\theta}_{it}, \varphi_{it}^0, x_{it}, u_{it}))$ . For notational simplicity, drop the conditioning variables and write  $p_{ijt}(\pi_{ijt}) = F_j^e(\pi_{ijt})$ . The expected payoffs from participation are given by:

$$\pi_{ijt} = \sum_{J^A \subseteq J_{it}^P} \prod_{\substack{k \in J^A \\ k \neq j}} p_{ikt}(\pi_{ikt}) \prod_{\substack{k \in J_{it}^P \setminus J^A \\ k \neq j}} (1 - p_{ikt}(\pi_{ikt})) \cdot \pi_j(\tilde{\theta}_{it}, \varphi_{it}^0, x_{it}, u_{it} | J^A) \equiv \Psi_j(\pi), \quad j \in J_{it}^P$$

---

<sup>5</sup>The expected payoff from participation also depends on the set of invited potential bidders  $J_{it}^I$ . However, since it is entirely determined by  $\tilde{\theta}_{it}$ , we omit it for notational simplicity.

where the summation goes over all possible sets of entrants  $J^A \subseteq J_{it}^P$ ,  $\pi_j(\tilde{\theta}_{it}, \varphi_{it}^0, x_{it}, u_{it} | J^A)$  is the expected payoff from bidding given the set  $J^A$  of actual bidders submitting their bids, and  $\pi = \{\pi_{ijt}\}_{k \in J_{it}^P}$  is a profile of expected profits from participation. Stacking all of the resulting equations, we obtain a fixed point characterization of the entry stage equilibrium:

$$\pi = \Psi(\pi) \tag{5}$$

Given the assumptions on the bidding stage, Brouwer's fixed point theorem guarantees the existence of equilibrium in the entry game.

Similarly to [Athey, Levin, and Seira \(2011\)](#); [Athey, Coey, and Levin \(2013\)](#), I assume a parametric model for entry. The probability that a potential bidder will enter the auction  $t$  is defined as:

$$p_{ijt} \equiv P[d_{ijt} = 1 | w_{ijt}] = \frac{\exp\{w_{ijt}\eta\}}{1 + \exp\{w_{ijt}\eta\}} \tag{6}$$

where  $w_{ijt}$  is a vector of state variables explaining participation depending on the profile of ex-ante bidder preference weights, initial fleet structure, and the observed and unobserved auction level heterogeneity  $w_{ijt} \equiv w(\tilde{\theta}_{it}, \varphi_{it}^0, x_{it}, u_{it})$ .  $\eta$  is a vector of parameters. Parametric specification reduces computation burden, especially with unobserved auction heterogeneity. However, it rules out a potential multiplicity of equilibria. Therefore, I assume that the same equilibrium generates all observations in the data.

Collect the bus producers who paid the entry costs and enter the bidding stage in the set  $J_{it}^A \subseteq J_{it}^P$ . If  $J_{it}^A = \emptyset$ , the auction concludes without a winner. The operator receives a payoff associated with an unsuccessful auction.

## 4.5 STAGE 3: BIDDING

After paying the entry cost, the actual bidders in the set  $J_{it}^A$  enter the bidding stage in which they simultaneously submit their sealed bids in a roughly standard independent private value sealed bid low-price auction game with favoritism.

**INFORMATION STRUCTURE.** There are three main pieces of information revealed to the actual bidders at the beginning of the bidding stage: the identities of all actual bidders

$j \in J_{it}^A$ , the individual total cost of delivering the object, and the realization of the profile of ex-post bidder preference weights that serve to rank the bids.

Project completion costs concern all the costs needed to physically produce and deliver the buses and include two elements. The main cost component is the expected (or objective) value of the order driven by the auction-level heterogeneity  $\Xi(x_{it}) \cdot u_{it}$ . There is also an idiosyncratic shock to the average cost  $c_{ijt}$  that reflects the individual variation in the project completion costs. This variation may come, among others, from differences in input prices faced by firms, contractual arrangements with contractors, and opportunity costs. Each actual bidder draws a private realization of the shock:

$$c_{ijt} \sim F^c(\cdot) : [\underline{c}, \bar{c}] \rightarrow [0, 1]$$

where  $F^c$  is a continuous cumulative distribution function with associated density  $f^c$  that is strictly positive on the entire support, and such that  $\frac{1-F^c(\cdot)}{f^c(\cdot)}$  is strictly decreasing. The draws of the individual aspect of the project completion cost are private information of each actual bidder. The functional form of the cost distribution is known to all players.

The total cost of producing and delivering the order by actual bidder  $j$  is given by:

$$\Xi(x_{it}) \cdot u_{it} \cdot c_{ijt}$$

The bidding depends also on the levels of favoritism used to rank the bids. Specifically, at the beginning of the bidding stage, an ex-post bidder preference weight for each actual bidder is drawn randomly from a distribution conditional on the respective ex-ante bidder preference weight chosen by the bus operator in the auction design stage:

$$\theta_{ijt} \sim F^\theta(\cdot | \tilde{\theta}_{ijt}), \quad j \in J_{it}^A \tag{7}$$

The resulting profile of ex-post bidder preference weights  $\theta_{it} = \{\theta_{ijt}\}_{j \in J_{it}^A}$  is announced publicly and observed by the econometrician. I assume that every actual bidder in every auction faces the same conditional distribution of ex-post bidder preference weights. This

implies that conditional on the choice of ex-ante bidder preference weights, the bidders are treated symmetrically.

**DECISION VARIABLES.** In the bidding stage, the actual bidders choose their bids. Denote the submitted bids by  $\{b_{ijt} : j \in J_{it}^A\}$ . Bidder  $j \in J_{it}^A$  wins if their bid is ranked the lowest after adjusting for the ex-post bidder preference weights:

$$j = \arg \min_{k \in J_{it}^A} \theta_k b_k$$

Lower values of  $\theta_j$  compared to the opponents allow actual bidder  $j$  to win despite submitting a higher bid. The ex-post bidder preference weights are unique up to normalization. Without loss of generality, I normalize the weight of the most preferred bidder to 1.

**OBJECTIVE FUNCTION.** In case of winning, an actual bidder  $j$  delivers the order. That means they must pay the project completion cost  $\Xi(x_{it}) \cdot u_{it} \cdot c_{ijt}$ . In return, they receive the payment of their submitted bid  $b_{ijt}$ . Auction losers receive payment of zero at the bidding stage (note they already paid the entry cost).

The actual bidders maximize the expected payoff from bidding:

$$b_{ijt} = \arg \max_{b \geq 0} \pi_j(b, c_{ijt}; \theta_{it}, x_{it}, u_{it}) = \\ \arg \max_{b \geq 0} \left( b - \Xi(x_{it}) \cdot u_{it} \cdot c_{ijt} \right) \cdot \text{Prob} \left[ \theta_{ijt} b \leq \theta_{ikt} b_{ikt} \text{ for all } k \neq j \in J_{it}^A | b, c_{ijt}; \theta_{it}, x_{it}, u_{it} \right]$$

**EQUILIBRIUM.** A Bayesian-Nash equilibrium in the bidding game is a set of optimal bidding functions:

$$\beta_j(\cdot | \theta_{it}, x_{it}, u_{it}) : [\underline{c}, \bar{c}] \rightarrow [b_j, \bar{b}_j], \quad j \in J_{it}^A$$

such that for each potential bidder  $j$ ,  $\beta_j(c_{ijt} | \theta_{it}, x_{it}, u_{it})$  is a best response at the individual cost realization  $c_{ijt}$  to the actions of other players, assuming they follow their equilibrium strategies. Equilibrium bidding is also a function of the profile of ex-post bidder preference weights  $\theta_{it}$ , the observed auction-level heterogeneity  $x_{it}$ , and the unobserved auction-level



heterogeneity  $u_{it}$ . Implicitly, it depends on the functional forms of the individual cost distributions<sup>6</sup>.

There exists a unique equilibrium of the bidding game, in which the optimal bidding functions are strictly increasing in  $c_{ijt}$ . The strategy of proving this claim relies on showing that each auction with bidder preference weights has a respective auction without bidder preference weights but an altered cost structure. The alternative auction is constructed on the so called *effective units*, including project completion costs  $\tilde{c}_{ijt} = \theta_{ijt}c_{ijt}$  and bids  $\tilde{b}_{ijt} = \theta_{ijt}b_{ijt}$ . Note that the distributions of the project completion costs in the alternative auction formulation, including their extremities, are those from the original rescaled by the ex-post bidder-preference weight  $\theta_{ijt}$  for every agent. Using the alternative formulation of the auction, we can invoke the result of [Lebrun \(2006\)](#), which proves the existence and uniqueness of equilibrium in a first-price independent values auction with a general cost structure. I describe this idea in detail in the online appendix [C](#).

Multiplicative separability in the project completion costs implies multiplicative separability in the bidding function ([Krasnokutskaya, 2011](#)):

$$\beta(c, u, \Xi) = \Xi \cdot u \cdot \beta(c, 1, 1) \text{ for any } \Xi, u \quad (8)$$

This result greatly simplifies the computation, as the optimal bidding function obtained for a given set of participants  $J^A$ , a profile of ex-post bidder preference weights  $\tilde{\theta}$ , and  $\Xi = u = 1$  can be used to generate the optimal bidding function for any values of  $u$  and  $\Xi$ .

CONNECTION WITH THE EXPECTED PROFITS AT THE ENTRY . Having derived the optimal bidding, we can provide the equations for the expected payoffs used in the previous stages. In particular, the expected profit from participation given the set of actual bidders  $J^A$  is:

$$\pi_j(\tilde{\theta}_{it}, x_{it}, u_{it} | J^A) = \int_{c_{ijt}} \int_{\theta_{it}} \pi_j(c_{ijt}; \theta_{it}, x_{it}, u_{it} | J^A) dF^\theta(\theta_{it} | \tilde{\theta}_{it}) dF^c(c_{ijt}) \quad (9)$$

---

<sup>6</sup>The optimal bidding naturally also depends on the set of actual bidders  $J_{it}^A$ . However, it is implied by the remaining arguments, so I skip it for notational simplicity.

## 4.6 CONCLUSION OF THE GAME

Once the winner is announced, the payoffs are realized, and the fleet gets updated with new buses delivered by the winner. The game iterates back to stage 0, where a new interaction between the operator and bus produces commences.

## 4.7 ADDITIONAL ASSUMPTIONS

**SINGLE BIDDER AUCTIONS.** The majority of auctions in my data observe only one actual bidder. Single-bidder auctions pose a challenge in the low-price auction literature. Theory predicts that a single bidder bids infinity because there is no competitive pressure from other participants. This is not a feature of the data. It may be optimal for sellers to keep bids relatively low despite being a single bidder. Setting unreasonably high prices may be a negative signal to the operators, discouraging them from setting up favorable bidder preference weights in auctions. A way to rationalize finite bids in single-bidder auctions is to introduce a binding reserve price. I follow the approach of [Li and Zheng \(2009\)](#) by assuming that it is a common belief among potential bidders that if they turn out to be a single bidder, they compete against the auctioneer drawing a project completion cost realization from the distribution  $F^R(\cdot)$ . Auctioneer's bids are effectively a secret reserve price, that is, a reserve price that is not revealed before bidding.

A proper choice of  $F^R(\cdot)$  is crucial for rationalizing single-seller bids and poses a challenge in a setting with bidder preference weights. Assuming that single bidders compete against the auctioneer who draws from the same cost distribution (as in [Li and Zheng \(2009\)](#)) would lead to a situation in which prices offered by single bidders are systematically lower than prices offered by the most preferred bidders facing low pressure from participating but discriminated competitors. To deal with this issue, I invoke results of [Guerre, Perrigne, and Vuong \(2000\)](#) stating that the distribution of costs can be uniquely derived from the distribution of bids. I assume the auctioneer draws their costs from a distribution generated by the bids submitted by under-preferred bidders  $G^R(\cdot)$ . The rationale behind this modeling choice is that discriminatory bidder preference weights assigned by a buyer indicate the acceptable price increase in case the most preferred bidder wins. In particular, I choose the

distribution of pseudo auctioneer’s bids in single bidder auctions  $G^R(\cdot)$  to be a distribution of bids submitted by bidders having bidder preference weights higher or equal to 5.5, which is the average bidder preference weight assigned to the second most preferred bidder in my data. The first order condition for optimal bidding is:

$$b_{ijt} - \Xi(x_{it}) \cdot u_{it} \cdot c_{ijt} = \frac{1 - G^R(b_{ijt})}{g^R(b_{ijt})} \quad (10)$$

where  $g^R(\cdot)$  is the density of pseudo auctioneer’s bids. This approach allows me to obtain realistic markups also in single bidder auctions.

OPERATOR’S ESTIMATE. The bus operators publish a non-binding estimate of the order costs after the bids are submitted but before they are opened. I assume it can be expressed as:

$$b_{i0t}^* = \Xi(x_{it}) \cdot u_{it} \cdot b_{i0t} \quad (11)$$

where  $b_{i0t}$  is an *iid* random component associated with the availability of funds or other aspects shaping the buyer’s cost estimate. The multiplicative separability allows me to use it as an additional source of information regarding the auction-level heterogeneity. The operator’s estimate plays a key role in the identification argument.

FURTHER REMARKS. Section D in the online appendix contains additional discussion of modeling choices.

## 5 ESTIMATION

This section describes the algorithm for estimating the model primitives from the data and presents the estimation results.

### 5.1 OVERVIEW

For each auction, the data is assumed to include the set of potential bidders  $J_{it}^P$ , a complete description of the initial fleet  $\varphi_{it}^0$ , the observed auction characteristics  $x_{it}$ , indicators of

auction participation  $d_{ijt}$ , the profile of ex-post bidder preference weights  $\theta_{it}$  (I use the observed weights for auction participants and the imputed weights for the non-participants), submitted bids  $b_{ijt}$ , and operator’s estimate of the costs of the order  $b_{i0t}$ .

The algorithm is sequential, which means that the estimates obtained in a previous step are used as inputs in the subsequent step. Table 2 lists the objects to be estimated in the proper order with a short description of the estimation approach.

Table 2: Model primitives to estimate

object	description	strategy
Stage 3: Bidding		
$\Xi(x_{it})$	the observed auction-level heterogeneity component	bid homogenization
$F^u(\cdot)$	distr. of the unobserved auction-level heterogeneity	non-parametric deconvolution
$F^c(\cdot)$	distr. of the individual project completion cost	non-parametric deconvolution
Stage 2: Entry		
$F^\theta(\cdot \tilde{\theta})$	conditional distribution of ex-post preference weights	latent class model
$\tilde{\theta}_{it}$	the realizations of ex-ante preference weights	latent class model
$p_{ijt}(w_{it})$	the conditional probabilities of participation	integrated maximum likelihood
$F_j^e(\cdot)$	the distribution of entry costs	bounds by participation profits
Stage 1: Auction design		
$\alpha, \gamma_{ij}$	parameters of operator’s utility	random coefficient multinomial logit

The estimation departs from the bidding stage. Relying on the multiplicative separability, I regress the logarithms of the operator’s estimates (equation 11) and the logarithms of submitted bids (equation 8) on the observed auction-level characteristics to obtain estimates of the observed auction-level heterogeneity component to the project completion cost  $\Xi(x_{it})$ . This is the bid homogenization method by Haile, Hong, and Shum (2003). Residuals from the regression form pairs of sums of the unobserved auction-level heterogeneity and idiosyncratic components of the operator’s estimate and bids, respectively. I use these pairs to estimate the distribution of unobserved auction-level heterogeneity  $F^u(\cdot)$  by applying non-parametric deconvolution methods (Kotlarski, 1966; Li and Vuong, 1998; Krasnokutskaya, 2011; An-

[dreyanov and Caoui, 2022](#)). I also retrieve the distribution of  $G^R(\cdot)$  of pseudo auctioneer’s bids in single bidder auctions justifying bidding in single bidder auctions in a similar fashion on the subsample of bids submitted by under-preferred bidders (with  $\theta_{ijt} \geq 5.5$ ). With this estimate in hand, use the first order condition for optimal bidding in the single-bidder auctions (equation 10)—in which single bidders bid against the pseudo auctioneer’s bids—to obtain the distribution of the individual component to the project completion cost  $F^c(\cdot)$ . This concludes the estimation of the bidding game.

In the next step, I move to the entry game. The conditional distributions of ex-post bidder preference weights (equation 7) are estimated using a latent class mixture model, assuming a parametric family for  $F^\theta(\cdot|\tilde{\theta})$ . I choose Gamma distributions because they easily match the domain of ex-post bidder preference weights and enable more efficient numerical integration of the expected profits (equation 9) through generalized Laguerre quadrature. I allow the mixing probabilities to depend on the initial fleet structure  $\varphi_{it}^0$  and the order characteristics  $x_{it}$  and impute the realizations of ex-ante bidder preference weights  $\tilde{\theta}_{it}$  by assigning a value associated with the highest mixing probability for each bidder and every auction. Now, I am ready to solve for optimal bidding and retrieve the expected bidders’ profits, expected bids, and winning probabilities conditional on participation. Solving for the equilibrium bidding function is difficult because virtually every auction in the data is equivalent to a first price auction with arbitrary cost distributions: with varying extremities of the cost domain. I extend the shooting algorithm for solving for optimal bidding using [Lebrun \(2006\)](#) characterization of equilibrium in such auctions. The equilibrium objects are then numerically integrated over the distribution of the idiosyncratic component to the project completion cost distribution and the joint distribution of ex-post bidder preference weights (equation 9). Since the optimal bidding preserves the multiplicative separability of the project completion costs, I need to numerically solve for the optimal bidding only once for each combination of participants and a profile of ex-post bidder preference weights. I also estimate parametric conditional probabilities of participation (equation 6) by maximizing the integrated (with respect to the unobserved auction-level heterogeneity term) likelihood function. Having the expected profits and entry probabilities, I derive bounds on the distribution of entry costs without having to solve for the fixed point of the entry game (equation 5) by

noticing that the realizations of entry costs must have been above and below the expected profits from participation among the non-participants and participants respectively.

Lastly, I proceed with the estimation of the auction design stage primitives. Using winning probabilities and expected winning bids, I form the expected utility from choosing a given profile of ex-ante bidder preference weights to the bus operator. The resulting model is a multinomial logit discrete choice model with random coefficients, which I estimate by maximizing the simulated likelihood function (based on equation 3).

The online appendix E presents additional discussion regarding the estimation strategy.

## 5.2 RESULTS

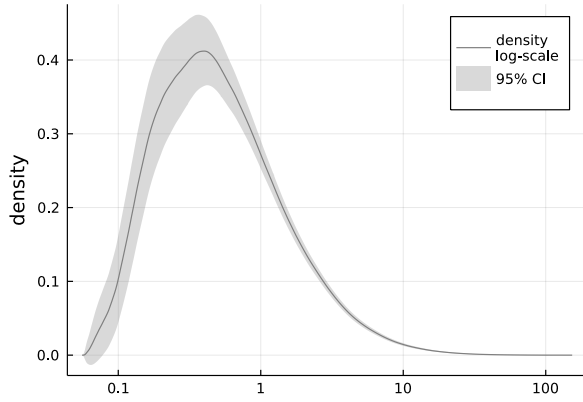
Unless stated differently, all monetary terms are expressed in millions of 2010 USD. Standard errors are obtained by bootstrap based on drawing operators' paths of auctions.

**BIDDING STAGE.** Project completion costs are a product of observed ( $\Xi$ ) and unobserved ( $u$ ) auction heterogeneity and individual component  $c$ . Figure (2) presents the estimated densities of these components.

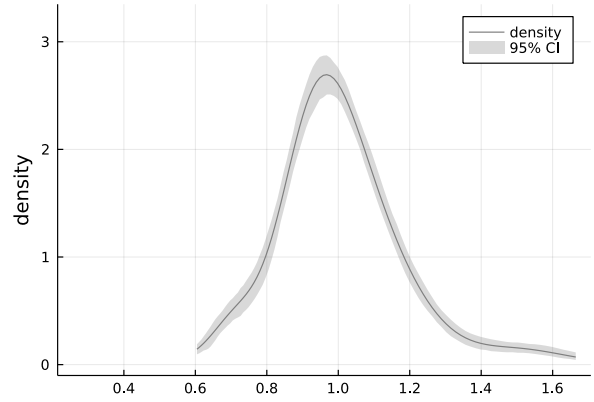
The distribution of observed auction heterogeneity is skewed with a long right tail, as depicted in panel (2a). Its variance is large compared to the mean (panel 2e). This suggests that the auction orders reflect diverse needs of the operators. The market observes a few outstandingly large contracts. The distribution of unobserved auction heterogeneity (panel 2b) is more symmetric than the observed heterogeneity component and also much more concentrated around the mean. However, the longer right tail indicates the presence of infrequent auctions with high realizations of  $u_{it}$ .

The observed and unobserved auction-level heterogeneity describe auction-level profitability. Since the observed heterogeneity realization is technically a fitted value in a regression in which the explanatory variable is price, I think of it in terms of the (average) monetary value of the project. The unobserved heterogeneity shifts it up or down due to factors hidden for the econometrician, including specification details not included in the sample or less tangible factors such as prestige of realizing contracts for big operators.

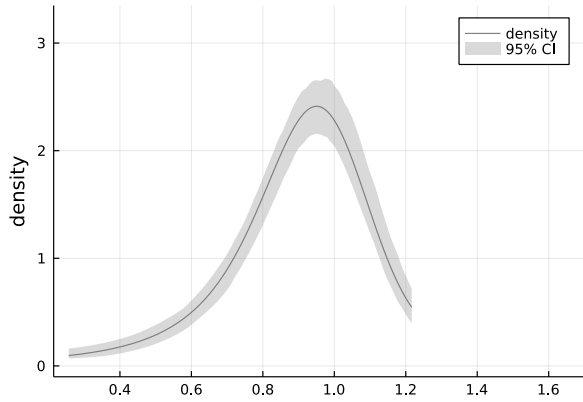
Figure 2: Estimated densities of project completion cost components.



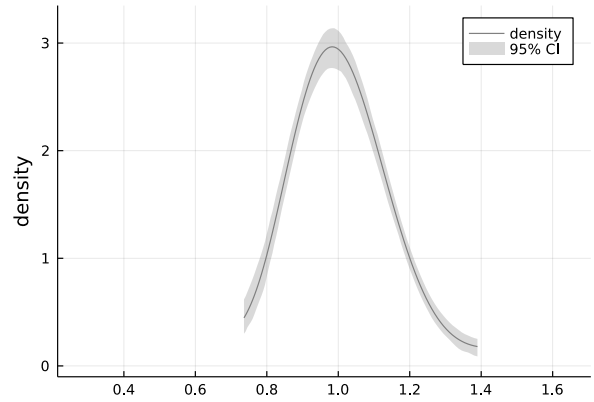
(a) observed auction heterogeneity  $\Xi(x_{it})$



(b) unobserved auction heterogeneity  $u_{it}$



(c) individual completion costs  $c_{ijt}$



(d) operator's price estimate  $b_{it}^0$

(e) descriptive statistics

	min	med	max	mean	st dev
observed heterogeneity (millions 2010 USD)	0.056	1.948	153.32	5.029	9.082
unobserved heterogeneity	0.604	0.99	1.665	1.008	0.179
price estimate	0.736	1.0	1.39	1.008	0.131
completion costs	0.257	0.916	1.217	0.89	0.185

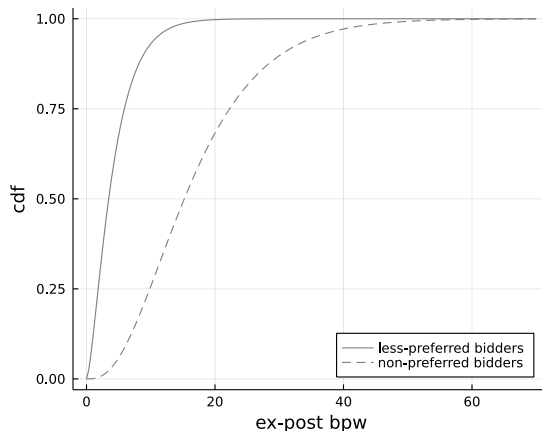
The lower bound of the individual component is nearly three times smaller than the lower bound of the distribution of the operator's price estimates (panel 2d). The latter is a good proxy for a variation in submitted bids, free from the effects of bidder preference weights and participation patterns. Bidders' opportunity for significant mark-ups is concentrated only in the low-density regions of the left tail (panel 2c). The difference in median bidders' individual project completion cost components and operator's price estimates amounts to approximately 10% (panel 2e) and remains comparable at the right tail.

EX-ANTE BIDDER PREFERENCE WEIGHTS. Figure (3) presents estimates of the ex-ante bidder preference weights distributions and associated latent class mixing weights. The upper part of panel (3a) shows the summary statistics of conditional distributions of ex-post bidder preference weights given ex-ante preferential status. The most preferred bidders are sure to keep this status if they decide to participate, regardless of the entry decisions of other bidders. The less preferred bidders receive ex-post bidder preference weight on average of 4.31. This number is comparable to the bidder preference weight of the second most preferred bidder observed in the raw data. Less-preferred bidders have a good chance of receiving favorable weights. Non-preferred bidders also have a chance for small draws, but it is slight. On average, they are assigned ex-post bidder preference weight exceeding 17.

Figure 3: Conditional distributions of ex-post bidder preference weights:  $F^\theta(\cdot|\tilde{\theta})$ .

latent class distributions			
bidders	mean	st. dev.	N
most-preferred ( $\tilde{\theta} = 0$ )	0	0	938
less-preferred ( $\tilde{\theta} = 1$ )	4.31	3.42	359
non-preferred ( $\tilde{\theta} = 2$ )	17.06	9.81	602

(a) parameter estimates



(b) cumulative distribution functions

These results should be interpreted considering that final preferential treatment depends on entry. If the most preferred bidder enters, they retain the most preferred status having their ex-post bidder preference weight equal to one with probability one. However, the preferential treatment of actual bidders that have been assigned a positive ex-ante bidder preference weight depends on the configuration of other participants and their draws of ex-post bidder preference weights.

I use estimates of the mixing weights associated with latent distribution to infer ex-ante bidder preference weights. The mixing weights describe the probability of being assigned  $\tilde{\theta}_{ijt} = 1$  instead of  $\tilde{\theta}_{ijt} = 2$ , given a set of covariates. I explain this probability using a

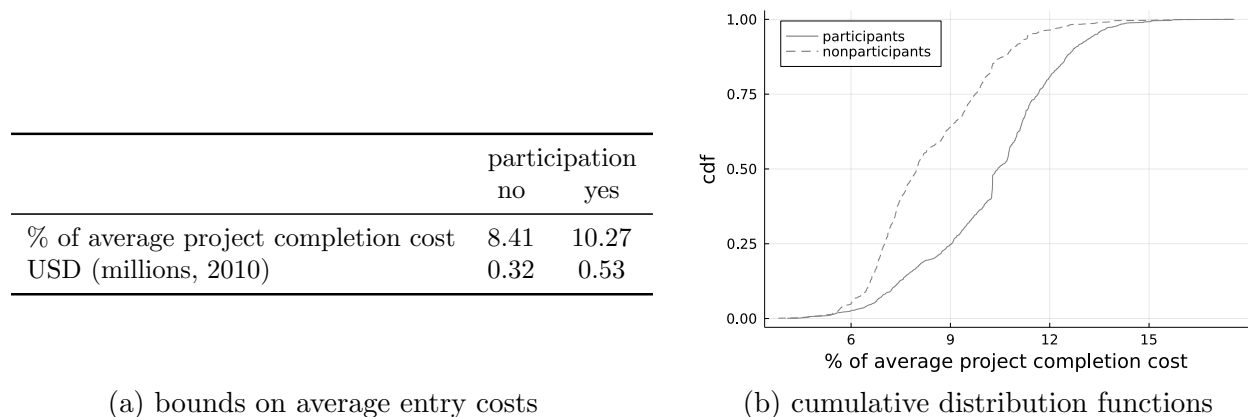


range of variables related to the operator’s fleet and operator and order characteristics<sup>7</sup>. The number of implied most preferred bidders is nearly equal to the number of auctions, confirming a substantial degree of favoritism.

Panel (3b) shows that the conditional distribution of ex-post bidder preference weights among non-preferred bidders first-order stochastically dominates the analogous distribution among less-preferred bidders. This feature has not been imposed on the estimation routine. Estimates recover assumed patterns, strengthening the distinction between less- and non-preferred potential bidders. That speaks in favor of latent class specification and estimation reliability.

ENTRY COSTS. The comparison between average expected profits among potential bidders who did not enter and the participating bidders reveals bounds on average entry costs. Figure (4) presents the results.

Figure 4: Expected profits from the participation in an auction.



Panel (4a) indicates that the average entry cost lies between 8.41% and 10.27% of the average project completion costs. These numbers are high but comparable with entry costs obtained in the literature (Li and Zheng, 2009; Athey, Levin, and Seira, 2011). Estimated bounds are relatively tight. Their difference amounts to approximately 1.86% of the average project completion cost. Panel (4b) presents the empirical cumulative distribution of expected profits from participation among participants and non-participants. The cumulative

<sup>7</sup>The estimates are presented in table (10) in the online appendix F.

distribution function of entry costs lies between these two curves. In particular, it suggests that entry costs are contained in an interval between 5% and 15% of average project completion costs. The additional discussion about entry, including the estimates of entry probability parameters, is presented in the online appendix [F](#).

**OPERATOR’S STAGE.** The estimated coefficients are expressed in utility units and cannot be directly interpreted. To be able to say more about the degree of switching costs, I first express them in monetary terms by dividing them by the estimate of the price coefficient. The resulting numbers are known as Willingness-To-Pay (WTP) in the discrete choice literature. Second, I divide them by the average value of the order as measured by auction level heterogeneity  $\Xi(x_t) \cdot u_t$ . Table (3) presents the results. Table (12) in the online appendix [F](#) presents the unscaled estimates of the operator’s utility parameters.

Table 3: Quantification of disruption costs: Willingness-To-Pay approach.

unification dimension	estimate
brand in the fleet	10.58** (3.84)
won within past 3 years	9.61*** (2.87)
producer’s bus of the same drive in the fleet	7.99* (3.5)

Average effects expressed in % of the order value  $\Xi(x_t) \cdot u_t$ . Delta method standard errors in parentheses. p. val:  
 \*\*\*  $\leq 0.001$ , \*\*  $\leq 0.005$ , \*  $\leq 0.01$ .

On average, the incumbent’s win is priced at 10.58% of the average order value, holding other factors constant. Operators are willing to surrender an additional 9.61% of the average order value to ensure unification within the bus drive. WTP for winning of a producer that won a recent auction amounts to 7.99% of the order value. That means operators are willing to pay, on average, up to 28.18% of the average order value for the win of the most suitable incumbent. The WTP for an incumbent win in terms of order value may seem high, but this magnitude partially results from the tendency to carry small and frequent orders.

## 6 COUNTERFACTUAL ANALYSIS

In this section, I use estimates of the structural model to study counterfactual scenarios. The primary motivation is to find ways to increase buyers' welfare. I show that bid preference programs can balance the trade-off if an auction attracts sufficiently many bidders, whereas forcibly promoting competition while ignoring the underlying lock-in relationship between buyers and incumbent sellers would lead to counter-productive outcomes.

### 6.1 SEPARATING THE EFFECTS OF DISCRIMINATION AND AUCTION PARTICIPATION

Will operators face lower prices if they set less discriminatory bidder preference weights holding participation fixed or if they attract more bidders holding discrimination fixed? In this section, I separate the effects of bidder preference weights and participation on prices using the bidding stage estimates. I focus on ex-post bidder preference weights to keep the analysis closer to the raw data evidence.

I investigate how the ex-post bidder preference weights affect producers' bidding. I consider a counterfactual scenario in which I increase the bidder's preference weight by a standard deviation for each auction and each actual bidder. I study how this affects the average bid of a bidder whose weight increased, the average bids of their competitors, and the average winning bid. In this exercise, I keep the participation fixed, considering solely participation patterns from auctions in my data. I restrict my attention to auctions with at least two actual bidders for meaningful analysis. The results are summarized in table (4)

Table 4: Effects of discrimination on average bidding.

	mean	st dev
affected bidder's bids	-6.339	0.318
others bidders' bids	1.5	0.738
winning bids	0.248	0.824

Change in average optimal bidding between data and counterfactual scenario as % of data values.

Considering a set of auctions in my sample, increasing bidder preference weight for one of the bidders affects mainly their bidding behavior. The affected bidder decreases their

bids by 6.3% on average. The competitors do not react strongly, increasing their bids only by 1.5%. The expected winning bid increases only by 0.25%. This is because the potential discounts resulting from the decrease in price by the affected bidder are annihilated by their decreased probability of winning. Holding participation fixed, the discrimination channel does not significantly affect the procurement costs but shifts contract allocation towards more preferred bidders.

In the second step, I switch my attention to the effects of participation. I consider a counterfactual scenario in which I assume an additional potential bidder decided to enter. I study how this affects the average prices of original participants and average winning bids. To keep the analysis more realistic, I assume that the additional entrant is the non-participant with the lowest (the most favorable) ex-post bidder preference weights in the data. Table (5) presents the result of this counterfactual exercise.

Table 5: Effects of participation on average bidding.

	mean	st dev
true participants' bids	-0.499	7.333
winning bids	-5.09	3.615

Change in average optimal bidding between data and counterfactual scenario as % of data values.

Given the sample, the presence of an additional actual bidder leads to a decrease in winning bids by 5.1%. The average effect of an additional participant on the bids of true participants is small. However, its large variance suggests heterogeneous impacts. This is consistent with intuition, as the additional participant's weight may have been much larger or much smaller than the ones of true participants in the counterfactual scenario.

Results of this counterfactual exercise suggest that the bid preference program itself does not have to be a source of a significant increase in the expected winning price. It rather reallocates the winning probabilities among bidders. In turn, participation is the leading margin of potential reductions in procurement costs. This indicates that if the auctioneer is able to attract a sufficient number of bidders, bid preference programs may be suitable for balancing the trade-off between price and disruption costs. Specifically, preferred bidders are more likely to win, whereas the competitive pressure from other auction participants would keep the prices low.

## 6.2 THE VALUE OF BID PREFERENCE

In this section, I use the full structural model to perform counterfactual exercises. I show that forcibly promoting competition while ignoring disruption costs may lead to counterproductive results. In turn, bid preference programs may allow operators to balance their trade-off between achieving low prices and avoiding switching costs if buyers can ensure sufficient participation.

Two crucial aspects of the economic environment in my application need to be accounted for in counterfactual analysis. First, even though the decision model is static, the winner's identity in the current auction affects future outcomes. Second, there is inherent uncertainty regarding the identity of the auction winner associated with project completion costs at the bidding stage, potential bidders' entry costs, and operators' shocks related to the choice of ex-ante bidder preference weights. To account for these features, I adopt a specific counterfactual analysis method. I focus my analysis on the paths of auctions. A path is an ordered series of auctions by an operator in which the fleet evolves according to the winners' identities. I maintain the order, timing, and characteristics of auctions from the data to account for factors related to the operator's demand for new buses that are not accommodated in my model, including the availability of funding, expiration of the current fleet, and technological progress (e.g., zero-emission drives). For each auction on a path, I obtain the expectation of the value of the operator's utility associated with the order, prices submitted by potential bidders, the number of actual bidders, and winning probabilities. I use the latter to draw the winner's identity and proceed to the next auction on the path. I simulate paths within an operator<sup>8</sup> and average them to get one path of expected outcomes per operator.

The baseline scenario assumes the original data environment: auctions with bid preference and estimated probabilities of entry. I compare the outcomes under the baseline scenario to outcomes generated in three counterfactual scenarios. First, I take away the possibility of favoring bidders by assuming that each auction is a low-price auction in which the cheapest bid wins. Comparing buyers' welfare attained in the low-price auction, and the baseline

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<sup>8</sup>For nearly 90% of the operators, I can simulate all counterfactual paths and obtain the probability of their occurrence. For the remaining operators—who organized many auctions during the observation window—I obtain the result by randomly drawing from the set of all possible paths.

scenarios describes the value of bid preference to the operators. Second, I maintain the possibility of using bid preference but assume all potential bidders participate in each auction. Analyzing the perfect participation scenario aims to verify whether the bid preference helps buyers balance their trade-off by allowing them to reallocate winning probabilities towards incumbent bidders while keeping the prices low. Third, I consider a combination of the two mentioned scenarios: low-price auctions with full participation.

I use compensating variation to study changes in operators’ welfare. Specifically, I ask how much money operators need to be given (or taken) to be indifferent between the baseline and counterfactual scenarios. To answer this question, I obtain the expected buyers’ utility under the baseline scenario  $EU_0$  and analogous expected utility under counterfactual scenarios  $EU_i$ ,  $i \in \{L, S, B\}$  where  $L$  stands for low price auctions,  $S$  for full participation, and  $B$  for both. The resulting compensating variation is given as:

$$CV_i = \frac{EU_i - EU_0}{\alpha}, i \in \{L, S, B\}$$

where  $\alpha$  is the coefficient at a price in the operator’s utility function.

Table 6: Counterfactual exercise—compensating variation and contributing factors.

		Scenario		
		low price auctions	perfect participation	both
compensating variation	mean	-0.56	0.2	-0.37
	% positive	3.63	52.76	14.62
$\Delta$ price	mean	-0.03	-0.37	-0.3
$\Delta$ # participants	mean	0.17	1.88	1.89
$\Delta$ # brands if fleet*	mean	0.35	0.43	0.62

Compensating variation and prices expressed in millions of 2010 USD.

Table 6 presents the results. The first row describes the calculated compensating variation. Switching an auction format to low-price auctions has a detrimental effect on operators’ welfare. On average, operators are willing to pay 0.56 million 2010 USD for being able to use bid preference in an auction. In only 3.6% auctions, the low-price setting would improve buyer’s welfare. In turn, the operators are willing to pay 0.2 million to attract all potential bidders while being able to favor some of them. Increasing participation while allowing

for auction favoritism improves welfare in more than half of the auctions. Switching to a low-price auction format and, at the same time, ensuring full participation hurts buyers' welfare.

Switching to a low-price auction format leads to a slight decrease in prices (\$30,000 per auction). The insufficient increase in auction participation drives this: on average, auctions under the low-price scenario observe only .17 actual bidders per auction more than in the baseline scenario. Removing favoritism leads to a relative increase in entry by non-incumbents. After four auctions, an average fleet under the low-price scenario contains 0.35 more brands.

Perfect participation by construction tackles the problem of insufficient entry. The decrease in expected prices is a magnitude larger than under low-price auctions and, in over 52% of cases, outweighs the increase in disruption costs. This is possible because the bid preference program allows the operators to re-allocate winning probability towards the incumbents despite increased entry. Without bid preference, the price decrease associated with increased participation does not offset the increase in switching cost, as depicted in the last column of table 6.

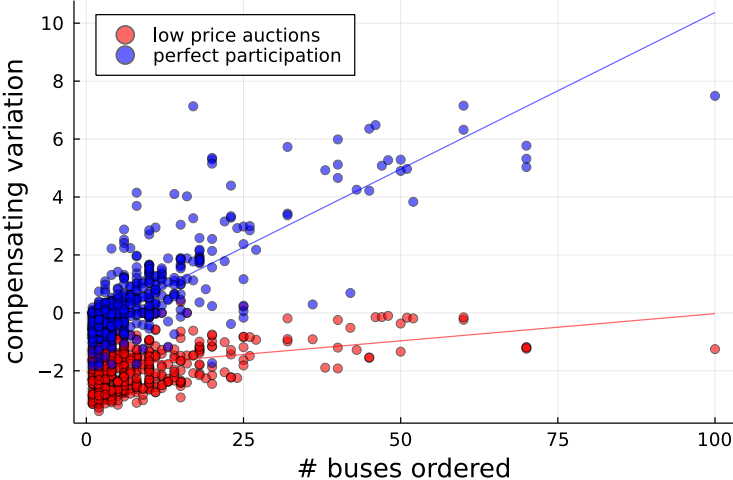


Figure 5: Compensating variation and order size. The solid line depicts a fitted trend.

The effects of switching to a low-price auction format and ensuring full participation are heterogeneous across auctions. Figure 5 shows the calculated compensating variation by the order size. In both scenarios, compensating variation is increasing in the number

of buses ordered in an auction. Intuitively, the significance of disruption costs decreases relative to the value of the order, being replaced by price as the primary contributor to welfare. The low-price auction scenario leads to small decreases in prices. Hence, the trend of compensation variation is flat. It would take orders of over 100 buses to make the price discount account for the increased costs of variety. In turn, with full participation, the price reduction is significant. On average, compensating variation is positive for orders of only five or more buses.

## 7 CONCLUSION

The results of this paper indicate the necessity of accounting for factors other than price in designing procurement markets. The welfare loss associated with eliminating auction favoritism is substantial. Potential bidder participation is key to improving the situation of the buyers and, hence, the efficiency of public spending. Bid preference programs may balance prices and switching costs successfully if the entry generates enough competitive pressure.

Low auction participation is associated with the lock-in relationship between an operator and a producer. The lock-in weakens participation incentives through two channels. First, the costs of variety induce operators to set up a system of preferential weights. Due to entry costs, non-preferred potential bidders do not find it profitable to participate in the auction. Second, incumbent advantage smooths out part of entry costs making them more likely to participate even without preferential treatment. In addition, participation is low because of the relatively small number of potential bidders across the auctions.

A few possible solutions could mitigate the lock-in effects and encourage more competition without sacrificing fleet unification motive and associated bid preference systems. First, the government may directly subsidize entry. The counterfactual scenario of full participation can be seen as a limiting case of such a subsidy. The idea is to pay a part of potential bidders' entry costs to encourage participation without affecting bid preference, especially among potential bidders with non-favorable weights and non-incumbents who cannot enjoy the benefits of incumbent advantage. However, it may be hard to implement an entry subsidy.



An immediate set of questions to ask is who to subsidize—should it be all potential bidders consistently with the principle of equal treatment of the bidders, or should it be potential bidders with non-favorable preference weights? A successful subsidy program would also have to be designed in a way to prevent fictitious entry in which potential bidders would participate only to collect the subsidy, without the intention to bid competitively and win the auction.

Another type of subsidy is to create favorable conditions encouraging more producers to enter the market and increase the pool of potential bidders. However, this type of subsidy faces the risk related to the fact that new producers would initially carry the non-incumbent status in all of the auctions, and it may be hard for them to establish their position on the market. As a result, they may be forced to exit the market, making the subsidy wasteful.

Subsidizing costs of participation in an auction or entry to the market effectively targets non-incumbent participation. Even though it is likely to decrease prices, it may hurt operators' fleet unification efforts and fail to improve market efficiency. Since the low number of potential bidders in auctions is partially related to the fact that producers specialize in specific types of buses, a more effective subsidy may be to support the development of a more diverse pool of products offered by existing producers. This subsidy would induce more competition among the incumbents, giving the promise of lower prices while keeping the pool of incumbent brands constant.

The results of the counterfactual analysis indicate that with an increase in order size, the relative importance of disruption costs decreases. Hence, a simple idea is to make operators organize auctions for larger orders, perhaps less frequently. To consider uncertainty regarding future funding and the fact that only a fraction of buses is to be replaced every year, the object of an auction could be the right to deliver buses within the next  $n$  orders, for some  $n > 1$ . Such an auction may be much larger than most of the auctions observed in my data, so it should attract more bidders regardless of incumbency status due to higher profitability. At the same time, the role of the costs of variety would be less critical relative to the value of the order. As a result, operators may lower the degree of discrimination, as price discounts related to more competitive settings may surpass the costs of introducing a new brand to their fleet. Less discrimination would imply even more actual bidders. Since its winner

would serve a few consecutive orders, the uncertainty regarding the winner’s identity would decrease compared to the baseline setting. Therefore, larger (but potentially less frequent orders) would mitigate the adverse effects of disruption costs and simultaneously encourage potential bidders’ participation.

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# APPENDIX

## A PROCURING CITY BUSES IN POLAND

### A.1 PRODUCERS' TECHNOLOGICAL SOLUTIONS – EXAMPLES

As specified in EU directives (2014/24/EU, 2014/25/EU), scoring auctions and associated scoring rules aim to take into account the quality of the goods procured in addition to their price and ensure, in this way, the best value for money purchases. Operators choose bus-related criteria to promote specific qualities, fulfilling lawmakers' intentions. However, in many cases, the solution receiving a full score may not be objectively better than non-preferred solutions. For example, windshield division is the most frequently used criterion related to bus technology. Bus windshields can be divided vertically into two halves, making replacement cheaper in case of one-sided damage. However, the dividing column may affect the driver's view. Panoramic (non-divided) windshields offer a better view. However, in case of any damage, replacement gets more costly. Operators assign a full score for both solutions, confirming that neither solution dominates the other. The possibility of applying windshield division is related to how the bus chassis is constructed. Some producers do not offer a divided windshield.

Another frequently used criterion assigns points for the engine's horizontal or vertical alignment. Using horizontal engines, producers can increase the number of passenger seats. In addition, necessary components may be more accessible. Vertical engines, in turn, decrease the number of passenger seats but usually allow for limiting the number of stairs passengers need to climb to take a seat. Again, the alignment of an engine is an essential part of a bus and cannot be easily adjusted. Depending on the operator, both solutions may be given a full score.

### A.2 A SIMPLE EXAMPLE OF AUCTION

To understand how the mechanism of assigning bidder preference weights through scoring criteria related to bus technological solutions works, consider a simple example of an auction

Table 7: Scoring auction design – example.

Auction Design			Bidders			
			S		M	
price	prop. to lowest	max points	value	points	value	points
price	prop. to lowest	80	\$150k	80	\$200k	60
windshield division	yes	5	✓	5	–	–
	no	0	–	–	✓	0
engine alignment	horizontal	15	–	–	✓	15
	vertical	0	✓	0	–	–
total		100		85		75

with three scoring criteria: price, windshield division, and engine alignment. The lowest price scores 80 points, and more expensive offers receive proportionally less. Buses with their windshields divided receive 5 points, and horizontal engine alignment is worth 15 points. Table (7) presents the details.

Suppose that two bidders submit their offers. S offers cheaper buses, hence gets 80 points in price criterion. Their buses have divided windshields and vertically aligned engines, which results in 5 points assigned for bus-related criteria. M submits a higher price and hence receives fewer points. They offer no windshield division but receive 15 points for a vertically aligned engine. Eventually, S wins with 85 assigned points.

In this auction, M receives more points than S in bus-related criteria. Hence, it is more preferred than S by the operator. If M submits a price lower than S, then they surely win. The link between points assigned by the scoring rule and bidder preference weights lies in the question of how much higher price M can submit compared to S’s bid and still win. Using the fact that points for price are assigned proportionally to the lowest submitted price, the necessary and sufficient condition for M to win despite bidding a higher price is:

$$\frac{b_S}{b_M} \cdot 80 + 15 \geq 80 + 5 \quad \Rightarrow \quad b_M \leq 1.143b_S$$

The second inequality corresponds precisely to how we define bidder preference weights  $\theta$ . The weight for the most preferred bidder M  $\theta_M = 1$ , and  $\theta_S = 1.143$ .



## B DATA COLLECTION AND PROCESSING

### B.1 DATA COLLECTION

The first step of data collection was to identify auctions. To produce a comprehensive picture of the market, I purchased a list of all auctions that have been concluded between 2011–2019 from a small consulting company operating in the industry. I expanded the list for 2006–2010 and 2020–2022 using Tenders Electronic Daily (TED), a European online service on public procurement<sup>9</sup>, tracking new arrivals in operators’ fleets, and browsing industry press<sup>10</sup>.

For each auction identified in the first step, I sought official documents to retrieve order requirements, scoring rules, and submitted bids. This data was still available online for more recent auctions. Others came from various sources. The consulting company mentioned previously has provided me with a significant fraction of the missing documents. I requested the remaining documents directly from the operators. However, some older auctions were not available any longer through this channel. I have managed to retrieve documents for some older auctions by scraping Internet archives (among others, Wayback Machine - an Internet archive services, [web.archive.org](http://web.archive.org)) and document sharing platforms ([docplayer.pl](http://docplayer.pl) was particularly helpful). In total, I collected documents for 85% of all identified auctions between 2006 and August 2022 (97% for auctions in 2011–2022). The auction data has allowed me to identify public bus operators whose fleets were subsequently scraped from <http://phototrans.eu/>.

### B.2 DATA PROCESSING

The official procurement documents come in the form of pdf files of varying quality. I process them using optical character recognition algorithms to retrieve auction order requirements. However, extracting the definition of criteria within the scoring rule was hard to automate, as each operator formulates them in their own way, using different expressions and formats. I process each auction manually to maximize the quality of extracted scoring rules, as they

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<sup>9</sup>European Law requires publishing a contract notice at TED if the estimated value of the contract exceeds a certain amount of money. That amounts to approximately 2–3 buses.

<sup>10</sup>Infobus ([transinfo.pl/infobus](http://transinfo.pl/infobus)) and Transport Publiczny ([transport-publiczny.pl](http://transport-publiczny.pl)) were particularly useful.

are of central interest to my analysis. For similar reasons, auction results (bidders, bids, final scores) have been processed manually as well.

I focus my attention on orders for low-entry or low-floor buses, as these constitute a standard in the European urban bus market and are nearly always demanded by municipal operators. Moreover, I restrict the sample to cover orders for buses of lengths exceeding 8 meters. This is motivated by the fact that the smallest vehicles are produced with different technology and mostly by different manufacturers. All of the potential bidders for which I impute the bidder preference weights offer buses only above 8 meters. These account for approximately 90% of all auctions (925).

### B.3 DATA IMPUTATION

The official documents provide the total number of points assigned by the scoring rule to participating bidders. This allows me to construct bidder preference weights among participants and use them to solve for optimal bidding. However, to investigate how bidder preference weights affect the bidder’s entry, I also need to know the number of points in criteria related to the bus that would be assigned to nonparticipants had they entered an auction. This requires knowledge of particular technological solutions used by producers. I learned them from a range of sources, including official product brochures, results of other auctions in which scoring criteria included a feature of interest and a given producer participated, internet galleries, and YouTube videos<sup>11</sup>, and use them to impute the number of points assigned by operators to the non-participating potential bidders.

The imputation proceeds in two steps. First, using knowledge of bus technological solutions by producers, scoring rules announced by operators, and sets of potential bidders across auctions, I develop an algorithm assigning points to the potential bidders. I focus on 65 bus characteristics used as technological criteria (referred to as processed criteria), covering 80% of the total number of points assigned in criteria related to the bus in all auctions and eight producers with 95.5% of sales in the auction data. For each auction, potential bidder, and processed scoring criterion, the algorithm chooses a solution offered by the producer that

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<sup>11</sup>For more observable characteristics, for instance, counting the number of seats available from the floor level that are easily accessible for disabled passengers.

would receive the highest number of points within the criterion. Next, the number of points assigned in this way across processed criteria is summed. If the scoring rule is constructed using only processed criteria, the algorithm returns the sum. If the scoring rule contains criteria outside of the set of 65 processed criteria, the algorithm rescales the obtained sum by the ratio of the total number of points available in all bus criteria to the total number of points available in processed criteria.

There are two key aspects of the imputation process. First, the algorithm needs to accurately assign points for technological solutions. For various reasons, producers may not choose to offer solutions that would bring them the maximum number of points within a given criterion<sup>12</sup>. To test the algorithm quality, I compare the true number of points received by auction participants within the processed criteria with the analogous sum imputed by the algorithm. Table (8) presents the results of the t-test with the null hypothesis of the same averages. A very large p-value suggests satisfactory quality of the imputation algorithm.

Table 8: Imputation algorithm quality test: comparison of the means.

true	imputed	difference	p-val
16.735 (9.613)	16.672 (9.623)	0.063	0.887

Second, the rescaling procedure implicitly assumes that the processed criteria are chosen at random from the set of all criteria related to bus technology. To test whether rescaling does not introduce bias into the final imputed number of points, I sum the true number of points received by auction participants within the processed criteria, rescale this number, and compare it to the true total number of points assigned to the participant. Table (9) presents the results. I cannot reject the null that rescaling is unbiased.

Table 9: Scaling quality test: comparison of the means.

true	scaled	difference	p-val
19.819 (9.615)	19.907 (9.806)	-0.088	0.875

<sup>12</sup>There may be a few reasons behind that. For example, the producer may offer a cheaper solution or offer a technological innovation that was not predicted by the algorithm.

In the last step, I use equation (1) to transform the total number of points assigned to the bidders in technological criteria into the bidder preference weights.

## C BIDDING EQUILIBRIUM

In what follows in this section, I skip the *it* notation for clarity of exposition.

### C.1 EXISTENCE AND UNIQUENESS

Any low-price sealed bid auction with bidder preference weights can be expressed as an asymmetric low-price sealed bid auction without bidder preference weights, defined on the so-called effective units. To see this, for all  $j$  define effective bids as  $\tilde{b}_j = \theta_j b_j$  and effective cost as  $\tilde{c}_j = \theta_j c_j$ . Note that now  $\tilde{b}_j \in [\theta_j b_j, \theta_j \bar{b}_j, ] \equiv [\tilde{b}_j, \bar{\tilde{b}}_j]$ . Analogously,  $\tilde{c}_j \in [\theta_j c_j, \theta_j \bar{c}_j, ] \equiv [\tilde{c}_j, \bar{\tilde{c}}_j]$ .

Define also functions  $\tilde{\gamma}_j : [\tilde{b}_j, \bar{\tilde{b}}_j] \rightarrow [\tilde{c}_j, \bar{\tilde{c}}_j]$  for each  $j$  in the following way:

$$\tilde{\gamma}_j(\tilde{b}_k) = \theta_j \gamma_j\left(\frac{\tilde{b}_k}{\theta_j}\right)$$

This function is the effective inverse cost function. Note that:

$$\tilde{\gamma}_j(\tilde{b}_j) = \theta_j \theta_j \gamma_j\left(\frac{\tilde{b}_j}{\theta_j}\right) = \theta_j \gamma_j\left(\frac{\theta_j b_j}{\theta_j}\right) = \theta_j \gamma_j(b_j) = \theta_j c_j = \tilde{c}_j$$

Eventually, let  $\tilde{F}_j(\tilde{c}_j) = Prob[\tilde{C}_j \leq \tilde{c}_j] = Prob[\theta_j C_j \leq \theta_j c_j] = F_j(c_j)$ . Then  $\tilde{F} : [\tilde{c}_j, \bar{\tilde{c}}_j] \rightarrow [0, 1]$  is a well-defined cumulative distribution function of the distribution of effective costs  $\tilde{c}_j$ .

We can express the actual bidder  $j$ 's profit in terms of the effective units:

$$\begin{aligned} \pi_j(b_j; c_j, \theta, J^A) &= (b_j - c_j) \cdot \prod_{k \neq j} \left[ 1 - F_k\left(\gamma_k\left(\frac{\theta_j b_j}{\theta_k}\right)\right) \right] = \frac{1}{\theta_j} (\tilde{b}_j - \tilde{c}_j) \cdot \prod_{k \neq j} \left[ 1 - \tilde{F}_k(\tilde{\gamma}_k(\tilde{b}_j)) \right] \\ &\equiv \tilde{\pi}_j(\tilde{b}_j; \tilde{c}_j, J^A) \end{aligned}$$

This equation shows that the low-price auction with bidder preference weights can be expressed in terms of an equivalent standard asymmetric auction, defined on the effective

units. In particular, there is a 1-1 mapping between the inverse bidding functions and, thus, equilibrium bidding. This allows us to invoke theoretical results by [Lebrun \(1999, 2006\)](#) who proved the existence and uniqueness of the equilibrium in IPV asymmetric auctions with possibly different supports of bidders' cost distributions. Notably, the equilibrium is in pure strategies.

An alternative way of proving existence would be to show that auctions in my setting satisfy conditions in [Reny and Zamir \(2004\)](#). However, the way of proving proposition 1 suggested in this paper provides a more intuitive way to understand the mechanisms behind existence in this particular case. Focusing on the specific problem also delivers a characterization of the boundary conditions, which is essential for empirical work. Notably, the classic results of [Maskin and Riley \(2000, 2003\)](#) are insufficient for the proof because they assume the common lower extremity of the support of project completion cost distributions. The mapping from auctions with bidder preference weights into alternative auctions without them shifts the support of project completion cost distributions. Therefore, even assuming common support of project completion costs across bidders would not be enough to rely on Maskin and Riley's work to prove the proposition.

[Lebrun \(2006\)](#) provides also a characterization of the equilibrium.

## D DISCUSSION ON MODELING CHOICES AND ASSUMPTIONS

### D.1 ASSUMPTIONS ON AUCTION GAME

Entry and bidding stages build on the approach developed by [Krasnokutskaya and Seim \(2011\)](#) and [Athey, Levin, and Seira \(2011\)](#). That encompasses three major assumptions. First, the project completion cost is independent of entry cost and is revealed only for bidders who have decided to participate in the bidding and paid their entry cost, as in [Levin and Smith \(1994\)](#). This assumption is standard in the empirical literature and allows me to separate entry and bidding. It greatly reduces the computational burden, which is particularly important in my application, given arbitrary patterns of bidder preference weights.

Second, actual bidders know the identities of their opponents while submitting their bids. The market is small, with less than 10 regular bidders who repeatedly participate in auctions. It is likely that producers are well informed about each other, and so can accurately predict the identities of competitors. Moreover, potential bidders often engage in a public dialog with the operator regarding order characteristics between the auction announcement and the bid submission deadline. It frequently concerns specific technological solutions. Therefore, even though the identity of producers asking questions is kept secret, it is straightforward to infer who expresses interest in the bidding and how likely their entry is given the operator's response to their questions.

Third, entry and project completion cost realizations are bidders' private knowledge and are drawn independently across bidders from publicly known distributions  $F^e(\cdot|w_{it})$  and  $F^c(\cdot)$ , respectively. This assumption places my auction framework within independent private value paradigm. Entry costs are related to preparing the offer and may be a significant part of the final bids<sup>13</sup>. In my application, they refer to processing hundreds of pages of specifications, customizing products to fit the specification, making necessary arrangements with potential contractors, and properly quoting the product. Idiosyncratic variation in entry costs is attributed to differences in individual cost of labor (necessary to prepare an offer) or relation with the financial sector (proofs of financial capacity to realize the order). In turn, project completion costs concern all the costs needed to physically produce and deliver the buses. Order characteristics are specified in a detailed way so that bidders can predict their costs accurately. Therefore, existing variation in bids is associated with random variation in project completion costs. This variation comes from differences in input prices faced by firms, contractual arrangements with contractors, and opportunity costs.

## D.2 MYOPIC AGENTS

Agents make myopic decisions in each auction. This assumption is equivalent to setting a discount factor in a dynamic game to zero. Although one may argue it rules out some possibly

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<sup>13</sup>Li and Zheng (2009) find that on average they amount to 8% of the winning bid in the highway moving auctions, Athey, Coey, and Levin (2013) report average entry costs to be approximately 9% of the average bid in case of timber sales.

interesting inter-temporal strategies, it seems to describe the city bus market in Poland accurately. Market conditions evolve in a non-stationary way, which makes it difficult for agents to form rational expectations. Considering producers who participated in at least 10 auctions, four producers exited and five producers entered throughout the sample window. Moreover, technology is developing fast. In the early 2020s, electric buses accounted for most of the sales, as they received priority in dividing government funds, and nationwide legislation required operators to increase the fraction of zero-emission buses in their fleets. This reality would sound unlikely in the early 2010s when ON-drive buses dominated the market, and electric-drive buses were still in a conceptual development phase. Uncertainty increases as new technology stimulates entry, and incumbent producers adopt it at a different pace. A non-stationary environment also makes it difficult for the producers to form rational expectations, particularly in predicting future demand and competitors.

Nevertheless, the dynamics is implicitly accounted for. The fleet updates after every auction event, affecting the choices of both operators and producers. In particular, the operator's utility depends on the future fleet composition (which is contingent on the identity of the winner), so do the participation probabilities (hence entry costs).

### D.3 RANDOMNESS IN BIDDER PREFERENCE WEIGHTS

Distinguishing between bidder preference weights chosen by the operator and used to rank bids at the bidding stage is motivated by how bus operators choose to favor or discriminate against bidders. They assign bidder preference weights to potential bidders implicitly, through scoring rule criteria related to technological solutions. Many of these technological solutions are naturally fixed, for example, construction materials. Others may be subject to some minor innovations. For instance, a producer may introduce a new type of higher-capacity battery for electric drive. Sometimes the exact number of points a bidder would get is hard to predict. Gas consumption depends on the total weight of the bus, which in turn depends on a range of additional factors, including passenger information system, air conditioning system, etc. Moreover, some scoring criteria assign points to a bidder depending on the values of other offers. To account for these factors, I think of ex-post bidder preference

weights as random variables that depend on ex-ante bidder preference weights in a stochastic way.

## E DETAILS OF THE ESTIMATION

### E.1 OBSERVED HETEROGENEITY

Project completion cost  $c_{jt}^*$  of an actual bidder  $j$  is multiplicatively separable in observed and unobserved auction heterogeneity and private information of the bidder:

$$c_{ijt}^* = \exp\{\Xi(x_{it})\} \cdot u_{it} \cdot c_{ijt}$$

where  $\Xi(\cdot)$  is a function known up to a set of parameters. I assume that observed heterogeneity is linear in order characteristics:  $\Xi(x_{it}) = x_{it}'\xi$ .

Project completion cost separability implies that optimal bidding is also multiplicatively separable (Haile, Hong, and Shum, 2003; Krasnokutskaya, 2011):

$$b_{ijt}^* = \exp\{\Xi(x_{it})\} \cdot u_{it} \cdot b_{ijt}$$

where  $b_{ijt}$  is the component related to strategic bidding. It is a function of a private project completion cost draw  $c_{ijt}$ , hence also a random variable.

I also assume that the operator's cost estimate is also separable and can be expressed as:

$$b_{i0t}^* = \exp\{\Xi(x_{it})\} \cdot u_{it} \cdot b_{i0t}$$

where  $b_{it}^0$  is a random term related to funds availability, drawn independently across auctions according to a distribution function  $F^R(\cdot)$ .  $b_{i0t}$  is independent from  $u_{it}$ , mean independent from  $x_{it}$ , ex-ante and ex-post bidder preference weights, and  $\mathbb{E}[\log b_{i0t}] = 0$ .

Multiplicative separability simplifies the computation of the equilibrium bidding function across auctions. In particular,  $\beta(c, u, \Xi) = \Xi \cdot u \cdot \beta(c, 1, 1)$  for any  $u$  and  $\Xi$ .



In the first step, I separate observed auction level heterogeneity from the bids by applying bid homogenization (Haile, Hong, and Shum, 2003). Observed heterogeneity is linear in order characteristics:  $\Xi(x_{it}) = x'_{it}\xi$ . The multiplicative structure of bids allows me to write them as:

$$\log b_{ijt}^* = x'_{it}\xi + \log u_{it} + \log b_{ijt}, \quad j = 0, 1, \dots \quad (12)$$

where  $\xi$  is a vector of parameters.

$x_{it}$  includes the year, drive, length, size of the order, leasing indicators, additional items required in the order (e.g., electric battery chargers), delivery deadline, and length of warranties required. In addition, operators tend to have their standards regarding additional equipment, like types of driver's compartment, passenger information systems, or air conditioning. I include operator's fixed effects for operators with a total of 10 or more observations available. I add operator size to the specification as a proxy for financial capabilities and an indicator for European Funds. The remaining term  $u_{it}$  summarizes factors unaccounted for at the auction level, including deviations from the operator's standard requirements or prestige gains to the producer of selling to large and recognized operators.

I estimate the parameters of the equation (12) taking into account potential endogeneity resulting from the existence of unobserved fixed effect  $\log u_{it}$  at the auction level. Observed order characteristics  $x_{it}$  may be endogenous in equation (12). If they are correlated with either  $u_{it}$  or  $b_{ijt}$ , the standard estimation procedure would lead to biased estimates of  $\xi$  as well as other parameters in subsequent steps of estimation which rely on homogenized bids. By assumption,  $u_{it}$  is drawn independently from  $x_{it}$ . Problems may arise with  $b_{ijt}$ , as it is likely correlated with  $x_{it}$  both when  $j = 0$  (operator cost estimates) and  $j > 0$  (strategic components of bids).

The strategic components of bids  $b_{ijt}$  for  $j \geq 1$  are functions of project completions cost realization  $c_{ijt}$ , the configuration of actual bidders  $J_{it}^A$  and the profile of ex-post bidder preference weights  $\theta_{it}$ . The two latter objects are also dependent on ex-ante bidder preference weights  $\tilde{\theta}_{it}$ .  $c_{ijt}$  is uncorrelated with  $x_{it}$  by assumption. However, it is likely the case that more profitable auctions, as described by  $x_{it}$ , would attract more actual bidders. In turn, the operator may set more discriminatory ex-ante bidder preference weights when the order

is small to avoid fleet fragmentation. With more discriminatory bidder preference weights, the operator may expect higher prices and adjust their cost estimate.

In order to avoid endogeneity bias in estimates of  $\xi$ , I control for flexible functions of ex-post bidder preference weights, the number of actual bidders and dummy indicator for  $j = 0$  (denoted by  $m_{ijt}$ ), and order characteristics. Equation (12) becomes:

$$\log b_{ijt}^* = x_t' \xi + (m_{ijt} \cdot x_{it})' \nu + \log u_{it} + \epsilon_{ijt}^b, \quad j = 0, 1, \dots \quad (13)$$

where  $\xi$  and  $\nu$  are vectors of parameters and  $\epsilon_{ijt}^b$  is an error term. I estimate equation (13) using OLS on pooled data containing bids as well as cost estimates  $b_{i0t}$ . Eventually, I obtain homogenized bids as:

$$\log \hat{b}_{ijt} = \log b_{ijt}^* - x_{it} \cdot \hat{\xi} \equiv \log u_{it} + \log b_{ijt}, \quad j = 0, 1, \dots \quad (14)$$

where  $\hat{\xi}$  is a vector of estimated parameters  $\xi$ . In my application, homogenized bids are sums of unobserved heterogeneity realization and strategic bidding component (or cost estimate draw if  $j = 0$ ).

## E.2 UNOBSERVED HETEROGENEITY & PROJECT COMPLETION COSTS

Homogenized bids combine unobserved heterogeneity and the strategic part of bids. For each auction  $it$ , equations (15) for  $j = 0, 1, \dots$  can be interpreted as repeated measurements within a measurement error model, in which noisy observed value of  $\log \hat{b}_{ijt}$  contains a fixed element  $\log u_{it}$  and a random noise  $\log b_{ijt}$  that are additively separable and independent. This formulation allows me to invoke a statistical result of [Kotlarski \(1966\)](#), who showed that the characteristic function of a sum of two independent random variables is equal to the product of their characteristic functions. Based on this insight, [Krasnokutskaya \(2011\)](#) shows that distributions of unobserved heterogeneity and strategic components of bids are nonparametrically identified from a joint distribution of pairs of repeated measurements and proposes non-parametric estimators. I adapt her framework to the environment with endogenous entry and bidder preference weights.

Guerre, Perrigne, and Vuong (2000) show that the distribution of project completion costs  $c_{ijt}$  is identified from distribution of strategic component of the bids  $b_{ijt}$ . To recover the distribution of project completion costs in a setting with entry and bidder preference weights, it suffices to have an estimate of the distribution of a strategic component of the bids in one configuration of actual bidders and preference weights. The main difficulty in my application lies in the fact that the distribution of the strategic component of bidder's  $j$  bid  $b_{ijt}$  varies with the set of actual bidders  $J_{it}^A$  and the profile of ex-post bidder preference weights  $\theta_{it}$ . In most applications, the latter is drawn from a continuous distribution, which makes it nearly impossible to construct a sample of measurement pairs in which the noise factors follow the same distribution using exclusively  $\log b_{ijt}$ 's. The only exceptions are single bidder auctions, in which  $\log b_{ijt}$  depends only on the project completion cost realization. I pair them with the operator's price estimate realization  $b_{i0t}$  and estimate the distribution of  $b_{ijt}$ 's on the subsample of single bidder auctions. By assumption, single bidders compete against an auctioneer who draws from cost distribution implied by a distribution of non-preferred bidders  $G^R$ . I estimate this distribution in a similar fashion. The underlying project completion cost distribution satisfies:

$$c = b - \frac{1 - G^R(b)}{g^R(b)}, \quad b \sim G^1(b), \quad F^c(c) = G^1(b)$$

The distribution of unobserved heterogeneity cannot be recovered in the same step, as it is likely to affect entry. Instead, I estimate it on a pooled sample of all bids in all auctions with measurement pairs defined in the same way  $(b_{i0t}, b_{ijt})$ <sup>14</sup>.

Mean independence assumption on the distribution of  $b_{i0t}$  ensures that estimated distributions of  $u_{it}$  and  $b_{ijt}$  are expressed in the same units regardless of which subsample we use in estimation.

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<sup>14</sup>One may point out that I don't have repeated measurement for auctions with zero bidders. If actual bidders self-select based on values of  $u_{it}$ , missing measurements would cause an identification problem in estimating the distribution of unobserved heterogeneity. However, operators tend to announce new auctions for the same order soon after realizing the previous one attracted no bidders. These auctions are usually exactly the same, operators frequently reuse the same specification. Hence, I assume that in the repeated auction, the draw of  $u_{it}$  is the same in both the original and repeated auction. I impute measurements from repeated auctions to the original to control for unobserved heterogeneity in zero-bidder auctions.

### E.2.1 TECHNICALITIES OF ESTIMATION

The residualized bids combine unobserved heterogeneity and the strategic part of bids:

$$\log bu_{ijt} \equiv \log b_{ijt}^* - \Xi(x_{it}) = \log u_{it} + \log b_{ijt}, \quad j = 0, 1, \dots, J_{it}^P \quad (15)$$

In my application, I focus on pairs containing one measurement based on the operator's cost estimate and one measurement related to a submitted bid:

$$\begin{cases} \log bu_{i0t} = \log u_{it} + \log b_{i0t} \\ \log bu_{ijt} = \log u_{it} + \log b_{ijt} \end{cases}$$

for some  $j$ .

The estimation follows the nonparametric method in [Krasnokutskaya \(2011\)](#).

### E.3 EX-ANTE BIDDER PREFERENCE WEIGHTS

The most preferred bidders in terms of ex-ante bidder preference weights are sure to enjoy the most preferred position also at the bidding. Hence, whenever the ex-post bidder preference weight indicates the most preferred position  $\theta_{ijt} = 0$ , it follows that the ex-ante bidder preference weight does the same:  $\tilde{\theta}_{ijt} = 0$ . If  $\theta_{ijt} > 0$ , it could be that  $\tilde{\theta}_{ijt} = 1$  or  $\tilde{\theta}_{ijt} = 2$ . Therefore, observations of  $\theta_{ijt}$  are random draws from a mixture of distributions with two components  $F^\theta(\cdot | \tilde{\theta}_{ijt} = 1)$  and  $F^\theta(\cdot | \tilde{\theta}_{ijt} = 2)$  and mixing weights  $\text{Prob}[\tilde{\theta}_{ijt} = 1 | z_{ijt}^\theta]$  and  $\text{Prob}[\tilde{\theta}_{ijt} = 2 | z_{ijt}^\theta]$  respectively. Mixing weights denote probability of observing  $\theta_{ijt}$  given that it has been drawn from  $F^\theta(\cdot | \tilde{\theta}_{ijt} = 1)$  and  $F^\theta(\cdot | \tilde{\theta}_{ijt} = 2)$  respectively. They may depend on a vector of exogenous covariates  $z_{ijt}^\theta$ .

Since the draws of ex-post bidder preference weights are mutually independent, the problem boils down to a standard latent class model with two latent classes. Observation  $ijt$ 's contribution to the likelihood can be expressed as:

$$\ell(\theta_{ijt}) = \sum_{c=1}^2 \text{Prob}[\tilde{\theta}_{ijt} = c | z_{ijt}^\theta] \cdot f_j^\theta(\theta_{ijt} | \tilde{\theta}_{ijt} = c)$$

By assumption,  $f^\theta(\theta_{ijt}|\tilde{\theta}_{ijt} = c)$  are densities of Gamma distribution parametrized by  $(\sigma_{1c}, \sigma_{2c})$  for  $c \in \{1, 2\}$ . I also assume that the mixing probabilities can be written as:

$$\text{Prob}[\tilde{\theta}_{ijt} = 1|z_{ijt}^\theta] = \frac{\exp\{z_{ijt}^\theta\sigma_3\}}{1 + \exp\{z_{ijt}^\theta\sigma_3\}}, \quad \text{Prob}[\tilde{\theta}_{ijt} = 2|z_{ijt}^\theta] = \frac{1}{1 + \exp\{z_{ijt}^\theta\sigma_3\}}$$

where  $\sigma_3$  is a vector of parameters. I estimate  $\sigma = \{\sigma_1, \sigma_2, \sigma_3\}$  using maximum likelihood methods.

$z_{ijt}^\theta$ 's are not necessary to identify the conditional distributions of ex-post bidder preference weights but help to predict which observations in the data come from which conditional distribution. In my application,  $z_{ijt}^\theta$ 's contain a series of variables describing  $j$ 's contribution to the operator's fleet, including indicators of being in the fleet, past wins, past purchases as second-hand vehicles, and overall discrimination potential of an auction. I impute the ex-ante bidder preference weights using estimated mixing weights:

$$\tilde{\theta}_{ijt} = \arg \max_{c \in \{1, 2\}} \text{Prob}[\tilde{\theta}_{ijt} = c|z_{ijt}^\theta]$$

Without additional variation in  $z_{ijt}^\theta$ , the estimated mixing probabilities would be constant across the sample, making it difficult to impute ex-ante bidder preference weights.

## E.4 PARTICIPATION PROBABILITIES

Participation probabilities are expressed as:

$$P[d_{ijt} = 1|z_{ijt}, u_{it}; \zeta] = \frac{\exp\{z_{ijt}\zeta\}}{1 + \exp\{z_{ijt}\zeta\}}$$

where  $z_{ijt}$  is a vector of covariates,  $\zeta$  is a vector of parameters. We expect that auction profitability as expressed by auction level heterogeneity  $\Xi(x_{it}) \cdot u_{it}$  may affect entry. Hence, it is a part of  $z_{ijt}$ . The auction heterogeneity term creates a challenge in estimation, as it is observed by potential bidders deciding whether to participate in an auction but is typically unobserved by an econometrician.

Fortunately, the data contains useful information regarding  $u_{it}$ . Its distribution is recovered in one of the previous steps. Moreover, homogenized bids contain repeated (but noisy) measurements on realizations of the unobserved auction heterogeneity. Taking advantage of the fact that the support of all the components of homogenized bids is bounded and bounds have already been estimated, I derive bounds for each realization of  $u_{it}$  in the sample. Let  $\{\bar{u}, \underline{u}\}$ ,  $\{\bar{b}, \underline{b}\}$  and  $\{\bar{b}_0, \underline{b}_0\}$  denote bounds of the support of unobserved heterogeneity  $u$ , strategic bid components  $b$  and operator's price estimate  $b_0$ . For each submitted homogenized bid  $bu_{ijt}$  we can retrieve bounds for the realization of  $u_{it}$ :

$$u_{it} \in [\max\{bu_{ijt} - \bar{b}, \underline{u}\}, \min\{bu_{ijt} - \underline{b}, \bar{u}\}], \quad j \in J_{it}^A$$

Analogously, for the homogenized operator's cost estimate, it follows that:

$$u_{it} \in [\max\{bu_{i0t} - \bar{b}_0, \underline{u}\}, \min\{bu_{i0t} - \underline{b}_0, \bar{u}\}]$$

Intersecting all these intervals for repeated homogenized measurements within an auction returns the interval in which the unknown realization of  $u_{it}$  falls in. Denote it by  $[\underline{u}_{it}, \bar{u}_{it}]$ . Using this information, for each auction  $it$  I consider the conditional distributions of  $u$ , given the  $u \in [\underline{u}_{it}, \bar{u}_{it}]$ :

$$F^u(u | u \in [\underline{u}_{it}, \bar{u}_{it}]) = \frac{F^u(u) - F^u(\underline{u}_{it})}{F^u(\bar{u}_{it}) - F^u(\underline{u}_{it})} \equiv F_{it}^u(u)$$

with associated density  $f_t^u(u) = \frac{\partial F_{it}^u(u)}{\partial u}(u)$ .

Integrating over unobserved heterogeneity and making use of the fact that entry costs are independent across auctions and bidders, the log-likelihood function can be expressed as:

$$\begin{aligned} \ell(\zeta) = & \sum_{it} \sum_j \mathbf{1}[d_{ijt} = 1] \log \left( \int_u \mathbf{P}[d_{ijt} = 1 | z_{ijt}, u_{it}; \zeta] dF_{it}^u(u) \right) \\ & + \mathbf{1}[d_{ijt} = 0] \log \left( \int_u (1 - \mathbf{P}[d_{ijt} = 1 | z_{ijt}, u_{it}; \zeta]) dF_{it}^u(u) \right) \end{aligned}$$

which is maximized over  $\zeta$  to obtain the desired parameters. The covariates  $z_{ijt}$  include variables relevant for entry decision, including the observed auction heterogeneity  $\Xi_{it}$ , own ex-ante bidder preference weight, and ex-ante bidder preference weights of other potential

bidders. I also include a dummy for incumbent status to investigate potential incumbent advantage.

## E.5 ENTRY COSTS

Given the reduced form model of participation probabilities, I cannot recover the distribution of entry costs directly. However, the threshold rule is informative about bounds on realizations of entry costs. Specifically, the expected profits of potential bidders who did not enter must lie below their realization of  $e_{ijt}$ . Conversely, the expected profits of participants exceed their entry cost draw.

Obtaining expected profits  $\pi_{ijt}(\tilde{\theta}_{it}, J_{it}^P, x_{it}, u_{it})$  is a numerically challenging task, as they integrate out within-auction profits over a project completion cost draw  $c_{ijt}$ , vector of ex-post bidder preference weights  $\theta_{it}$  conditionally on a vector of ex-ante bidder preference weights  $\tilde{\theta}_{it}$ , and the set of actual bidders  $J^A$  conditionally the set of potential bidders  $J_{it}^P$ . Integration over the conditional distributions of ex-ante bidder preference weight is multi-dimensional. To maximize the accuracy of numerical integration and limit the number of necessary function evaluations, I use generalized Laguerre quadrature which fits particularly well to the framework with  $F^\theta$  parametrized as Gamma.

### E.5.1 SHOOTING ALGORITHM IN AUCTIONS WITH BPW

In order to calculate expected profits, I need to recover the set of optimal bidding functions  $\beta_j(c; \theta_{it}, x_{it}, u_{it}, J^A)$  for each auction, configuration of actual bidders, and vector of ex-post bidder preference weights. Equilibrium bidding is a solution to a set of ordinary differential equations that form a two-point boundary problem in which the boundary is known only on one side. In addition, the problem is singular at the known boundary. No closed-form solution exists. Numerical tools are needed to solve for equilibrium bidding.

I adapt a standard approach in solving for equilibrium bidding in asymmetric auctions based on shooting algorithms<sup>15</sup> to a setting with arbitrary bidder preference weights. Shoot-

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<sup>15</sup>The use of shooting algorithms in solving for auction equilibrium has been pioneered by [Marshall, Meurer, Richard, and Stromquist \(1994\)](#) and has been further extended by among others [Bajari \(2001\)](#); [Li and Riley \(2007\)](#); [Gayle and Richard \(2008\)](#). For a review, see [Hubbard and Paarsch \(2014\)](#).

ing algorithms are iterative methods of solving boundary value problems that exploit the fact that the solution is fully determined by  $\underline{b}$ , the lowest bid submitted in equilibrium. In a standard setting  $\underline{b}$  is the lowest bid submitted by all bidders. An additional challenge with bidder preference weights comes from the fact that the lowest bid submitted by a given bidder is not necessarily the lowest bid submitted in equilibrium in general,  $\underline{b}$ . Intuitively, discriminatory bidder preference weight may imply that the probability of winning at the lowest realizations of costs is very low, and the under-preferred bidder is actually better off raising their lower bound of the support of bids submitted in equilibrium. This phenomenon is known as bid separation (Hubbard and Kirkegaard, 2019)<sup>16</sup>. Lebrun (2006) showed that regardless of bid separation, the lowest possible bid submitted in equilibrium  $\underline{b}$  still uniquely defines the equilibrium. That makes shooting algorithms particularly suitable for settings with arbitrary bidder preference weights. Guided by Lebrun’s characterization of the equilibrium in a general setting, I adapt the shooting algorithm to account for potential bid bifurcation points<sup>17</sup>.

Hubbard and Kirkegaard (2019) solve for equilibrium bidding with bid separation using methods based on polynomial approximations. However, they consider a special case in which there is a good candidate for a bifurcation point. In a slightly different setting, Bolotnyy and Vasserman (2023) use shooting algorithms accounting for possible bid separation. Both papers consider setting symmetric equilibria with two types of bidders in which only one bifurcation point can occur. To the best of my knowledge, no other empirical papers mention the possibility of bid bifurcation in their application.

Some authors express concerns about the instability of shooting algorithms, particularly in the neighborhood of singularity, that is, at the right boundary of project completion costs distribution (Bajari, 2001; Li and Riley, 2007). Fibich and Gavish (2011) provided a theoretical argument for the fact that the instability is not caused by a choice of the algorithm but rather a feature of the problem itself. They show that shooting methods perform worse with an increase in the number of bidders. In my application, the number of potential bidders is relatively low and I find the performance of shooting methods satisfactory.

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<sup>16</sup>These authors emphasize the importance of accounting for bid separation in solving for optimal bidding, showing in examples that a failure to account for it leads to wrong conclusions.

<sup>17</sup>With  $N$  different bidder preference weights, we may have up to  $N - 2$  bifurcation points.



## E.6 OPERATOR’S PROBLEM

The definition of the operator’s problem leads to a multinomial choice framework. This case is special compared to other models in the literature because the operator essentially chooses a distribution of winning probabilities and expected prices over a range of potential bidders instead of choosing a product itself. I follow the strategy of [Petrin \(2002\)](#) and [Gentzkow \(2007\)](#) to use a rich specification that would free the model from heavy dependence on the *logit* idiosyncratic taste shock associated with choices of  $\tilde{\theta}_{it}$ .

In the baseline specification, I assume that operator’s stream of utility associated with the win of producer  $j$  depends on a dummy indicating whether  $j$ ’s products are already in the fleet, a dummy of whether  $j$  has won an auction within the past 3 years, a dummy of whether  $j$ ’s buses have arrived as second-hand buses within last three years, indicator of the fact that the current generation of  $j$ ’s products is already in the fleet, indicator of whether  $j$ ’s buses of the same drive are already in the fleet and producer’s share in the fleet given their win. I also include producers’ fixed effects, which I model as random coefficients to account for unobserved qualities of a match in operator-producer pairs  $ij$ . The specification of random coefficients is standard: I assume they follow a normal distribution with mean and variance to be estimated. I follow the simulated maximum likelihood strategy, averaging over draws of random tastes and unobserved auction-level heterogeneity.

## F ADDITIONAL PARAMETER ESTIMATES

### F.1 LATENT CLASS MIXING WEIGHTS

### F.2 ENTRY PROBABILITIES

Table [\(11\)](#) presents estimates of the auction participation probability. To make the results interpretable, I focus on the average marginal effects (continuous variables) and average marginal changes (discrete variables) of explanatory variables on participation probability.

Table 10: Parameter estimates of  $\sigma_3$ —mixing weights of latent class distributions

mixing weights parameters $\sigma_3$	
incumbent	1.18*** (.24)
brand's current generation in the fleet	.444 (.29)
won within past year	.936** (.469)
2nd hand delivery within past year	.21 (.409)
won within past 3 yrs	.824** (.354)
2nd hand delivery within past 3 yrs	-.56 (.428)
producer's share in fleet if wins	-1.4*** (.425)
order size (log)	.497*** (.085)
ordered drive already in the fleet	-.52** (.219)
EU funds	.253 (.182)
compatibility index (non-incumbents)	1.32** (.667)
constant	-1.6*** (.404)

Standard errors in parentheses. p. val: \*\*\*  $\leq 0.01$ , \*\*  $\leq 0.05$ , \*  $\leq 0.1$ .

I distinguish three main channels affecting bidders' decision to enter: factors related to the auction competitive environment set by the operator, factors related to the operator's fleet, and factors related to characteristics of the order.

Potential bidders consider both their own and competitors' ex-ante bidder preference weights. Potential bidders who are less preferred ( $\tilde{\theta} = 1$ ) participate in an auction with a probability of 15.9 percentage points smaller than their most preferred competitors. This effect strengthens among the non-preferred potential bidders, whose entry rate is lower by 24.6 percentage points. Ex-ante bidder preference weights assigned to the competitors are also important. Intuitively, less preferable treatment of other potential bidders decreases competitive pressure and increases the chances of winning at the bidding stage. As a result, potential bidders whose most preferred opponent received  $\tilde{\theta} = 1$  participate with 11.5 percentage points higher probability than if facing opponents with  $\tilde{\theta} = 0$ . Those who are to compete against solely non-preferred bidders participate with a probability larger by 29.4 percentage points. Conditional on own and competitors' ex-ante bidder preference weights,

Table 11: Entry stage estimates – average effects on participation probability.

	average effects
competitive environment	
own weight: $\tilde{\theta} = 1$	−0.159*** (0.003)
own weight: $\tilde{\theta} = 2$	−0.246*** (0.015)
competitors' weights: 1st best $\tilde{\theta} = 1$	0.115*** (0.003)
competitors' weights: 1st best $\tilde{\theta} = 2$	0.294*** (0.003)
competitors' weights: 2nd best $\tilde{\theta} = 1$	0.023*** (0.004)
competitors' weights: 2nd best $\tilde{\theta} = 2$	−0.019*** (0.005)
competitors' weights: 3rd best $\tilde{\theta} = 1$	0.088*** (0.002)
competitors' weights: 3rd best $\tilde{\theta} = 2$	0.047*** (0.008)
# potential bidders	−0.035 (0.092)
fleet composition	
incumbent	−0.003 (0.005)
won within past 3 years	0.182*** (0.007)
2nd hand delivery within past 3 years	0.004* (0.002)
brand's current generation in the fleet	0.196*** (0.006)
producer's bus of the same drive in the fleet	−0.008 (0.004)
compatibility index (non-incumbents)	−0.01 (0.009)
# brand in the fleet	−0.014 (0.115)
characteristics of the order	
ordered drive already in the fleet	−0.095*** (0.011)
auction profitability $\Xi(x_t) \cdot u_t$	0.032 (1.089)
N	3214

The table presents average marginal effects for continuous variables, expressed in standard deviation units or marginal change effects for categorical variables. Delta method standard errors in parentheses. p. val: \*\*\*  $\leq 0.001$ , \*\*  $\leq 0.005$ , \*  $\leq 0.01$ .

the number of potential bidders does not play a significant role in deciding whether to participate in the auction.

The compatibility index between a non-incumbent producer and the operator's fleet is constructed as follows. First, I calculate correlation coefficients between the number of buses

in operators' fleets by all pairs of producers and average it over time. The compatibility index for a non-incumbent is an average of the correlations between the non-incumbent and incumbents, weighted by the fleet share of the incumbents. The compatibility index is, by definition, normalized to the interval  $[-1, 1]$ . High index values indicate high levels of compatibility between non-incumbent and the operator's fleet.

Conditional on the competitive environment, the operator's fleet structure explains significantly potential bidders' auction participation patterns. This can be interpreted as evidence of incumbent advantage among potential bidders. Incumbent advantage refers to a situation in which it is *easier* for incumbent potential bidders to participate in an auction. That may be related to the benefits of an established connection with the operator. With buses already in the operator's fleet, producers frequently have set up a network of authorized workshops near the operator's depot and spare parts delivery chains. These factors are often required in auction specifications and contribute to entry costs. Additionally, since operators tend to keep their standards roughly fixed across auctions, previous deliveries ensure that producers have already implemented these standards in their production lines.

Even though the incumbency status itself does not affect entry, producers who have delivered their buses recently are more likely to enter—by up to 18.2 percentage points if new buses have been delivered within the past three years. This finding supports the incumbent advantage hypothesis, as recent orders tend to correlate most with the current one. In the same spirit, auctions by operators with the newest generation of producer's products in their fleets are more likely to attract these producers. Incumbent advantage strengthens the lock-in between operators and producers.

Eventually, auction characteristics also affect entry. If the operator orders buses with a type of drive that has not been previously exploited, they may expect increased entry. Consistent with intuition, more profitable auctions attract more bidders; however, the estimate lacks precision.

### F.3 OPERATOR'S STAGE

Table (12) presents estimates of the operator's utility parameters. Consistent with descriptive evidence, operators favor incumbent producers. The preference towards winners of more

recent auctions is even stronger. Interestingly, a recent delivery of second-hand buses does not significantly improve producers' chances of being favored. This finding suggests that second-hand purchases are driven by factors other than purchases of new buses, such as availability. The lock-in relationship between operators and producers goes beyond the sole technical aspects of the buses, as suggested by a non-significant estimate at the dummy indicating that the current producer's generation of products is already in the operator's fleet. Not only do operators derive utility from overall fleet unification, but they also prefer the unification of sub-fleets defined by bus drives. Producer's share in the fleet, conditional on their win, increases the operator's utility, suggesting the existence of costs related to maintaining more diverse fleets. However, the coefficient is not precisely estimated.

Table 12: Estimates of operator's utility parameters.

	price
expected price	-2.106*** (0.364)
	fleet
brand in the fleet	3.506** (1.097)
won within past 3 years	3.184*** (0.754)
producer's bus of the same drive in the fleet	2.649* (1.09)
producer's share if fleet if wins	3.032* (1.518)
	brands
dummies	✓
random effects	✓
N buyers	176
N auctions	925
N alternatives	59428

Standard errors in parentheses. p. val: \*\*\*  $\leq 0.001$ , \*\*  $\leq 0.005$ , \*  $\leq 0.01$ . The specification with random coefficients will be added in the next draft. Current estimates of standard errors do not account for uncertainty related to using estimates obtained in the previous steps.